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# Industrial Transformation and Pollution Dynamics

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This paper constructs a multi-sector growth framework with heterogeneous pollution to study the joint dynamics of industrial upgrading and environmental quality. Industries differ in capital intensity and follow endogenous and finite life cycles consisting of expansion, peak activity, and decline, generating sequential structural transformation as the economy develops. Production generates two types of pollution: an industry-specific flow pollutant that vanishes upon exit, and a common stock pollutant that accumulates over time. Unlike existing EKC frameworks that treat pollution as homogeneous or abstract from industry turnover, the model shows that structural transformation fundamentally reshapes pollution dynamics through endogenous industry life cycles. The analysis shows that industry life cycles generate a sequence of transitional, trapezoidal Environmental Kuznets Curve (EKC) patterns at the sectoral level, while the common pollutant exhibits a single inverted-U-shaped EKC at the aggregate level. Policy counterfactuals reveal a capital-scarcity transmission mechanism through which policies that accelerate upgrading can raise the shadow price of capital, amplify pollution inflows, and delay environmental improvement. Implementing the socially optimal allocation requires a coordinated combination of policy instruments, and implementability constraints impose economically meaningful bounds on admissible policy parameters. These results provide a structural interpretation of EKC dynamics and clarify how policy and structural transformation jointly determine the co-evolution of economic development

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**Classification** F061.2; F061.3.

# 1 Introduction

The relationship between economic development and environmental quality has long been a central concern in environmental economics. A large empirical literature documents a non-monotonic relationship between income per capita and various measures of pollution, a pattern commonly summarized by the Environmental Kuznets Curve (EKC). While the precise shape and turning points differ across countries and pollutants, a recurrent stylized fact is that environmental pressure tends to intensify during early industrialization and decline only at more advanced stages of development.

A fundamental feature underlying this empirical regularity is that economic growth is rarely a purely intensive process. Development is typically accompanied by profound structural transformation. Industrial upgrading proceeds sequentially: new industries emerge, expand, and eventually decline as factor endowments evolve and feasibility constraints change. At the same time, pollution is inherently heterogeneous across production activities. Different stages of development are associated with different dominant pollution sources, reflecting shifts in industrial composition and production technologies. As a consequence, both the level and the composition of pollution evolve endogenously over the course of economic development.

Some pollutants are tightly linked to specific industries and disappear when those activities exit, while others accumulate over time and across sectors, exerting persistent environmental pressure. These observations suggest that environmental dynamics cannot be understood independently of structural change. Pollution should therefore be viewed as a multidimensional object whose structure co-evolves with industrial upgrading and capital accumulation.

Despite this close association, the theoretical foundations of the EKC remain incomplete. Existing theories typically attribute EKC dynamics to mechanisms such as rising demand for environmental quality, exogenous technological progress in abatement, or optimal environmental regulation. Production is often represented by a single sector or a highly aggregated structure, and pollution is commonly modeled as a representative pollutant. Changes in industrial composition and shifts in dominant pollution sources are therefore either absent or introduced in a reduced-form manner. The entry, expansion, and exit of industries are not modeled explicitly, and the evolution of pollution composition does not arise endogenously within a dynamic general-equilibrium framework.

This limitation is consequential. Without explicitly modeling industrial life cycles and pollution heterogeneity, it remains unclear whether EKC-type patterns are a fundamental consequence of structural transformation itself or instead depend on auxiliary assumptions such as exogenous technological trends or delayed policy responses. Moreover, the interaction between capital accumulation, industrial upgrading, and environmental dynamics remains largely unexplored. In particular, when upgrading toward more capital-intensive industries intensifies capital scarcity, the shadow value of capital may change in ways that feed back into pollution dynamics. Such feedback effects can alter not only the level of pollution but also the timing of its turning point.

This paper develops a dynamic multi-sector growth model in which industrial upgrading, capital accumulation, and environmental outcomes are jointly determined. Industries are characterized by finite life cycles consisting of growth, peak activity, and decline, reflecting endogenous changes in factor endowments and feasibility constraints. This life-cycle structure plays a central role in shaping both sectoral dynamics and aggregate outcomes, and implies that structural transformation alone may generate growth–stagnation cycles even in the absence of external shocks.

Building on this upgrading structure, the model distinguishes between industry-specific, non-accumulative pollution and a common, accumulative pollutant, and allows for endogenous investment in pollution-abatement technology. This distinction makes it possible to analyze how different types of pollution dominate at different stages of development and how the composition of pollution evolves endogenously as industries emerge and exit. Within this framework, EKC-type patterns arise as a direct consequence of structural transformation, rather than as a by-product of exogenous technological progress.

The model also uncovers a less explored transmission mechanism linking environmental policy to structural dynamics. Policies that alter the relative cost of pollution can accelerate industrial upgrading toward more capital-intensive sectors. When the aggregate capital stock is predetermined, such upgrading raises capital scarcity and its shadow price, which in turn feeds back into pollution accumulation through the pollution inflow term. As a result, environmental regulation may shift and re-time the pollution trajectory in ways that are not immediately intuitive. The interaction between capital scarcity, structural change, and pollution dynamics therefore plays a central role in shaping long-run environmental outcomes.

In addition to characterizing the social optimum, the paper studies its decentralized implementation. We introduce three policy instruments: a technology fund charge to finance improvements in common pollution-abatement technology, a capital income tax, and either Pigouvian taxes or Coasian permit quotas. We show that the joint use of these instruments supports a competitive equilibrium that replicates the socially optimal allocation, illustrating how environmental and capital taxation can be combined to address multiple distortions arising from structural transformation and pollution externalities.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 discusses several historical experiences that motivate the analysis. Section 4 presents the model. Section 5 characterizes the socially optimal allocation. Section 6 analyzes decentralized implementation.

Section 7 studies the resulting macroeconomic dynamics. Section 8 clarifies the distinction between time-based and income-based EKC. Section 9 reports numerical simulations. Section 10 presents counterfactual experiments. Section 11 concludes. Appendix I provides proofs of the main results, and the remaining appendices collect supplementary materials.

# 2 Literature Review

This paper relates to three closely connected strands of the literature: (i) structural change and industrial upgrading, (ii) EKC theories, and (iii) environmental policy and the internalization of pollution externalities.

## 2.1 Structural change and industrial upgrading

A large literature studies long-run economic growth in multi-sector economies, emphasizing structural change, sectoral reallocation, and industrial upgrading. A central insight of this literature is that persistent changes in production and expenditure structures can arise endogenously during economic development, even when aggregate growth follows a balanced growth path.

An important conceptual foundation is provided by the Schumpeterian view of growth. Aghion and Howitt (1992) develop a model of endogenous growth driven by creative destruction, in which economic expansion results from successive waves of innovation that replace older technologies with more advanced ones. Although their framework is not a multi-sector model in a narrow sense, it establishes a microfoundation for structural transformation driven by innovation, technological turnover, and the endogenous obsolescence of existing production structures.

Subsequent work demonstrates that structural change is compatible with balanced aggregate growth in fully specified multi-sector environments. From the demand side, Kongsamut, Rebelo, and Xie (2001) show that non-homothetic preferences generate systematic reallocation of consumption and production across sectors as income rises, even when aggregate variables grow at constant rates. Their analysis highlights how Engel curve effects alone can induce persistent changes in sectoral composition over the development process.

From the supply side, Acemoglu and Guerrieri (2008) show that capital deepening can generate non-balanced growth across sectors when production technologies differ in factor intensities or elasticities of substitution. In their framework, sectoral reallocation arises endogenously from technological heterogeneity, without relying on non-homothetic preferences. Complementing this perspective, Ngai and Pissarides (2007) develop a multi-sector growth model in which sectors differ in total factor productivity growth rates. They derive conditions under which systematic shifts in sectoral employment shares coexist with balanced aggregate growth, showing that labor reallocates toward sectors with relatively lower productivity growth. Together, these contributions formalize how general equilibrium interactions and technological differences generate structural change along a balanced growth path.

More recently, Herrendorf, Rogerson, and Valentinyi (2020) provide a unified analysis of structural change in both consumption and investment. Their framework emphasizes that sectoral reallocation occurs not only through changing consumption patterns but also through shifts in the composition of investment, highlighting the joint role of demand, capital accumulation, and sectoral technologies in shaping long-run development paths.

Within the framework of New Structural Economics, Ju, Lin, and Wang (2015) and Lin, Liu, and Zhang (2023) study industrial upgrading as an endogenous response to evolving factor endowments and technology choices along the comparative-advantage frontier. These studies emphasize the stage-dependent nature of development and the role of feasibility constraints and policy facilitation in shaping the sequence of industries that emerge, expand, and decline over time.

Overall, this literature establishes a rich set of mechanisms through which structural change and industrial upgrading arise endogenously during economic development. However, these studies abstract from pollution externalities and environmental constraints. As a result, they leave open how structural transformation interacts with environmental outcomes and how industrial upgrading shapes the evolution of pollution over the development process.

## 2.2 Environmental Kuznets Curve

A large body of literature studies the non-monotonic relationship between economic development and environmental degradation, commonly referred to as the Environmental Kuznets Curve (EKC). Early empirical work documents inverted-U-shaped relationships between pollution indicators and income per capita. Grossman and Krueger (1995) provide one of the earliest systematic empirical investigations of this relationship, laying the foundation for subsequent EKC research.

On the theoretical side, a variety of mechanisms have been proposed to account for EKC-type dynamics within growth models, each emphasizing a distinct channel through which pollution may eventually decline as income rises.

Stokey (1998) studies a dynamic AK framework that can be viewed as a Ramsey model augmented with pollution externality. In her setting, the EKC emerges from an intertemporal trade-off between capital accumulation and environmental quality: as income grows, the marginal utility cost of pollution rises, inducing a reallocation of resources toward pollution control. The non-monotonic pollution path therefore reflects preference-driven adjustments along a single-sector growth trajectory, rather than changes in production structure.

Andreoni and Levinson (2001) propose a highly parsimonious static framework in which pollution is jointly determined by consumption and abatement effort. Under increasing returns to abatement, optimal pollution is a non-monotonic function of income even in a static environment. In this case, the EKC arises purely from technological properties of abatement and income effects, without any dynamic accumulation, growth, or sectoral change.

Egli and Steger (2007) extend the static framework of Andreoni and Levinson (2001) to a dynamic growth setting. In a one-sector economy with linear technology, pollution arises as a by-product of consumption and can be reduced through abatement effort. Under suitable parametric conditions, optimal consumption and abatement policies imply a monotonic growth path of income and an inverted-U-shaped relationship between pollution and income. The resulting EKC is driven by the intertemporal trade-off between consumption, abatement, and capital accumulation along a single-sector growth trajectory, rather than by changes in production structure or sectoral composition.

Brock and Taylor (2010) develop the Green Solow model, in which pollution emissions are proportional to output while a constant fraction of production is devoted to abatement. In this framework, the EKC is generated by exogenous technological progress in abatement. As long as abatement technology improves sufficiently fast relative to population growth and output expansion, emissions intensity declines over time, eventually offsetting scale effects. The turning point of the EKC is thus pinned down by the relative growth rates of production and abatement efficiency, not by structural transformation.

Marsiglio, Ansuategi, and Gallastegui (2016) propose a growth model to study EKC dynamics through what they refer to as structural change. Although the model is described as a two-sector framework, it effectively features a single final-good sector using two accumulable inputs, manufacturing capital and services. Structural change is captured by an exogenous increase in the share of services in production, rather than by endogenous reallocation across sectors. Under certain conditions, this exogenous shift in factor shares can generate an inverted-U-shaped pollution path. While the model highlights a compositional channel linking economic structure and environmental outcomes, structural transformation is not endogenously determined, and sectoral entry, exit, or life cycles are absent.

Other contributions emphasize irreversibility, and regime changes. Prieur (2009) develops an overlapping generations model with irreversible pollution accumulation and limited natural assimilation capacity, and shows that in such environments the emergence of EKC is not guaranteed, highlighting how irreversibility can fundamentally alter pollution–growth dynamics. Boucekkine, Pommeret, and Prieur (2012) distinguish between technological and ecological switches, showing that EKC turning points can arise endogenously from discrete regime transitions rather than smooth adjustments.

To date, the theoretical literature has largely attributed EKC dynamics to mechanisms such as preference-driven demand for environmental quality, exogenous technological progress in pollution abatement, or increasing returns to abatement within highly aggregated production structures. In these explanations, structural change plays no explicit role. By contrast, Panayotou, Peterson, and Sachs (2000) provide compelling empirical evidence that EKC dynamics are closely associated with industrial composition effects, but stop short of offering a theoretical mechanism that links structural transformation to pollution dynamics.

### 2.3 Environmental policy and the internalization of pollution externalities

A third strand of the literature studies environmental policy instruments designed to internalize pollution externalities.

Sandmo (1975) provides the seminal general equilibrium analysis, showing that, under standard conditions, the optimal Pigouvian tax equals the marginal social damage generated by pollution. Zhang and Xu (2013) analyze the dynamic adjustment of carbon taxes in a policy-oriented model and show that the so-called green paradox does not arise. Golosov et al. (2014) develop a quantitative model of fossil fuel use and climate damages, deriving optimal carbon taxes that reflect both intertemporal damages and discounting.

Weitzman (1974) provides a classic comparison of price-based and quantity-based regulation, establishing a decision rule that characterizes the conditions under which taxes or quotas are welfare-dominant. Anderson and Duanmu (2025), working within an Arrow–Debreu general equilibrium framework, establish sufficient conditions for the existence and efficiency of equilibria under both emission taxes and emission quotas, and show that the two policy regimes are generally not equivalent.

A work most closely related to our paper is Acemoglu, Aghion, Bursztyn, and Hemous (2012), which develops a two-sector model of directed technical change with environmental concerns and shows how environmental policy can endogenously steer innovation toward cleaner or dirtier technologies. By embedding environmental externalities into a Schumpeterian growth framework, they demonstrate that policy interventions affect not only emissions outcomes but also the direction of technological progress and long-run growth. However, because the economy consists of only two sectors and pollution is represented by a single common pollutant, the model cannot capture the fact that different stages of economic development are typically associated with different types of pollution. As a result, it is not suited to analyze how the composition of pollution evolves over the development process.

# 3 Historical experiences

Historical experience suggests that pollution dynamics over the course of economic development are neither uniform nor driven by a single mechanism. As economies evolve, dominant industries, production technologies, and energy systems change sequentially, and the forms of pollution that prevail at each stage change accordingly. Pollution is therefore best understood as a stage-dependent outcome of structural transformation rather than as a stable by-product of aggregate growth. This section summarizes several recurring patterns, documented in existing studies and historical accounts, that motivate the multi-sector framework developed in this paper.

A first recurring pattern is that different stages of development are associated with different dominant forms of pollution within the same economy. Long-run data for China summarized by Zheng, Zhang, Zhu, and Zhao (2024) provide a particularly clear illustration. Specifically, Figure 1 in Zheng et al. (2024) plots the trajectories of multiple pollutants over the course of economic development and shows that, while many pollutants exhibit inverted-U-shaped dynamics, their peak timings differ substantially. Some pollutants reach their maximum intensity at relatively early stages of development, whereas others peak at intermediate or later stages. This staggered pattern appears within a single country over time, ruling out explanations based solely on geography or climate, and instead points to the evolving industrial structure as the underlying driver. The fact that different pollutants peak at different stages strongly suggests that they are associated with different dominant industries and production technologies that emerge and decline sequentially along the development path.

Historical experience from advanced economies reinforces this interpretation. In the United Kingdom, early industrialization in the nineteenth and early twentieth centuries was dominated by coal-based manufacturing and residential heating. During this phase, pollution was primarily characterized by smoke, soot, and sulfur dioxide emissions, giving rise to the well-known episodes of London smog. As coal-intensive industries expanded, these pollutants intensified; as coal use declined after the mid-twentieth century and cleaner energy sources were adopted, sulfur-based air pollution diminished sharply. The trajectory of coal-related pollution thus closely mirrored the rise and fall of coal-based industry.

As development proceeded, different forms of pollution became prominent. In the mid-twentieth century United States, rapid motorization and the expansion of petroleum-based activities generated severe photochemical pollution, particularly in urban areas such as Los Angeles. This pollution, driven by nitrogen oxides and volatile organic compounds, was qualitatively different from earlier coal-smoke pollution and reflected a new stage of development characterized by automobile dependence and suburbanization. Over time, as vehicle technologies improved and regulatory frameworks evolved, photochemical pollution declined. These episodes illustrate that industry-specific pollutants are historically contingent: they emerge, intensify, and recede together with the industries and technologies that generate them.

A second recurring pattern is that industry-specific pollutants tend to follow non-monotonic trajectories that mirror the life cycles of the associated industries. As industries enter and expand, their associated pollution rises; as they mature and eventually decline or are replaced, pollution falls. This rise–peak–decline pattern is visible in a wide range of historical contexts, including coal-related air pollution, industrial wastewater associated with heavy manufacturing, and vehicle-related urban air pollution. Such pollutants largely disappear once the corresponding industries lose economic significance, rather than persisting indefinitely at lower levels.

A third pattern concerns changes in the composition of pollution over the development process. Economic development involves systematic structural transformation, typically from agriculture to manufacturing and eventually toward services and more knowledge-intensive activities. As emphasized by Panayotou, Peterson, and Sachs (2000), this transformation alters not only the overall level of pollution but also its sources. Early stages of development are often dominated by pollution associated with heavy industry, basic infrastructure, and rapid urbanization, including local air pollutants, industrial wastewater, and solid waste. As these sectors decline in relative importance, such pollutants tend to diminish, while pollution associated with economy-wide energy use becomes increasingly prominent. Development therefore reshapes the structure of pollution, not merely its aggregate intensity.

Finally, historical experience highlights an important distinction between industry-specific pollutants and common, economy-wide pollutants. Carbon dioxide emissions provide the leading example of the latter. Unlike sulfur dioxide or industrial effluents, $CO_2$ is not tied to a single industry but is generated broadly across sectors through energy use. Its evolution reflects the aggregate outcome of overlapping industrial activities and energy systems rather than the life cycle of any particular industry. Over sufficiently long horizons, $CO_2$ emissions may also exhibit non-monotonic dynamics, but with turning points that typically occur later in the development process, consistent with its accumulative and economy-wide nature.

In brief, these historical experiences indicate that pollution dynamics are deeply intertwined with structural transformation. Economic development proceeds through successive stages characterized by different dominant industries, each associated with specific forms of pollution. Industry-specific pollutants follow the life cycles of the industries that generate them, producing transitional inverted-U-shaped paths, while common pollutants reflect the aggregate interaction of multiple industries and evolve over longer horizons. These patterns motivate a multi-sector framework in which pollution dynamics arise endogenously from industrial upgrading and changes in pollution composition over the course of economic development.

# 4 Model setup

Consider a closed economy under perfect competition, consisting of two types of sectors. The first type comprises a single industry producing a capital good, while the second type consists of a sequence of industries producing distinct consumption goods, indexed by $n = 0, 1, 2, \ldots$[1].

Throughout the paper, industries (and the corresponding goods) are ordered by their indices. Industries with larger indices are referred to as *higher* industries (or goods), while those with smaller indices are referred to as *lower* industries (or goods).

Assume that the production functions for each industry are given by

$$Y_0 = L_0, \quad Y_n = \min\left\{\frac{K_n}{a^n}, L_n\right\}, \quad n \in \mathbb{N}, \tag{1}$$

where $Y_n$ denotes the output of industry $n$, $K_n$ and $L_n$ are the capital and labor inputs used in industry $n$, $a > 1$ is a constant, and $\mathbb{N}$ denotes the set of non-negative integers. Here, $a^n$ measures the capital intensity of industry $n$. As $n$ increases, industries become more capital-intensive; accordingly, $a$ governs the rate at which capital intensity increases across industries.

[1] For simplicity, time dependence is suppressed whenever it does not cause confusion. In particular, a time-varying variable $x(t)$ is written simply as $x$, unless it is necessary to emphasize time. We use $\dot{x} = \frac{dx}{dt}$ to denote time derivatives. All variables are assumed to be non-negative; accordingly, non-negativity constraints are implicit in all optimization problems considered in this paper.

Assume that industry 0, a primitive agriculture, does not generate pollution, whereas each industry $n \geq 1$ produces two types of pollution. The first type is industry-specific and non-accumulative, affecting only the current time and dissipating afterward (e.g., noise pollution, light pollution). We refer to the pollution generated by industry $n$ as the $n$-th specific pollutant. The second type is common pollution produced by all industries and accumulative, primarily caused by capital utilization (e.g., air pollution, water pollution, carbon emissions). More specifically, common pollution is emitted during the production process and is also generated during the activities aimed at abating the specific pollution.

For simplicity, we assume that producing one unit of good $n$ generates $\theta^n$ units of industry-specific pollution, where $\theta \in (0, 1)$ is a constant. This formulation captures the idea that more capital-intensive industries are associated with lower pollution intensity[2]. We also assume that people take actions to reduce pollution, denoted as $Q_n$. And hence, for this specific pollution $P_n$, we have

$$P_n = \theta^n Y_n - Q_n. \tag{2}$$

We further assume that pollution abatement in industry $n$, denoted by $Q_n$, depends on both output $Y_n$ and abatement effort $A_n$. For simplicity, we specify

$$Q_n = (\theta\varepsilon)^n \min\left\{Y_n, \frac{A_n}{d^n}\right\}, \tag{3}$$

where $\varepsilon \in (0, 1)$ and $d > 1$ are constants. This specification captures the complementarity between production and abatement effort, as well as the increasing difficulty of pollution control in more capital-intensive industries. The parameter $\varepsilon$ governs the effectiveness of abatement for industry-specific pollution across industries.

[2]This assumption reflects a stylized negative relationship between capital intensity and pollution intensity. Capital-intensive industries typically employ more advanced production technologies and pollution-control equipment, which tend to reduce emissions per unit of output. While this relationship does not hold universally—particularly in some heavy industries—it is a reduced-form assumption in environmental and structural-change models and serves to capture technological upgrading in a parsimonious way.

Regarding common pollution $P$, people invest in pollution reduction efforts, leading to the following dynamics for $P$:

$$\dot{P} = MK - \delta P, \tag{4}$$

where $\delta > 0$ represents the self-cleaning strength of this type of pollution, $K$ is working capital used in production and industry-specific pollution abatement, and $M$ is the emission intensity (or pollution-generating coefficient), which is reduced by abatement-technology investment $A$:

$$\dot{M} = -AM. \tag{5}$$

Abatement-technology investment $A$ is financed out of the capital stock and enters the capital accumulation equation as:

$$\dot{Z} = \eta Z - \delta_1 K - \delta_2 A, \tag{6}$$

where $Z$ denotes the aggregate capital stock, and $\eta > 0$, $\delta_1, \delta_2 \in (0, 1]$ are constants. Capital is assumed to evolve according to an exogenous reduced-form regeneration process, with $\eta$ capturing the intrinsic growth rate of the capital stock. This process is taken to be independent of conventional market-mediated production activities.[3]

The parameters $\delta_1$ and $\delta_2$ represent the depreciation rates associated with the use of working capital and abatement-technology investment, respectively. Although $\delta_1$ and $\delta_2$ are allowed to be strictly less than one—so that only part of $K$ and $A$ is exhausted when they are employed—assuming full depreciation entails no loss of generality in our setting. Since depreciation rates are constant, they affect the model only through the capital accumulation equation. In

[3] This treatment adopts a reduced-form description of capital regeneration, abstracting from explicit market-based accumulation mechanisms. While learning-by-doing–based AK technologies, such as those discussed in Ju, Lin, and Wang (2015), provide one possible microfoundation, they typically generate additional positive externalities and thus require additional policy instruments. Since the focus of this paper lies elsewhere, we abstract from such mechanisms and model capital as a renewable input for simplicity.

particular, the rental price of capital already incorporates depreciation. Allowing for $\delta_1, \delta_2 \in (0,1)$ would therefore leave the qualitative results unchanged, at the cost of additional notation. For simplicity, we normalize $\delta_1 = \delta_2 = 1$ throughout.

There is a representative agent endowed initially with one unit of labor and capital $Z_0 > 0$. The instantaneous utility function is given by

$$U(C_0, C_n, P, P_n, n \geq 1) = \left( C_0 + \sum_{n=1}^{\infty} \kappa^n \left( C_n - \theta^{-n} P_n \right) \right)^{\alpha} - \sigma P, \tag{7}$$

where $\alpha \in (0,1)$, $\kappa > 1$, and $\sigma > 0$ are constants. Here, $C_n$ denotes consumption of good $n$, $P_n$ is the industry-specific pollution generated by industry $n$, and $P$ is the common pollution stock. Industry-specific pollution is weighted by $\theta^{-n}$, reflecting the agent's stronger aversion to pollution associated with higher goods. The disutility from common pollution is linear in $P$ with coefficient $\sigma$. The parameter $\alpha$ governs the curvature of utility with respect to net consumption, so that $1-\alpha$ represents the coefficient of relative risk aversion. Since $-U_P = \sigma$, the parameter $\sigma$ represents the marginal disutility of common pollution. Moreover, $U_{C_n}/U_{C_0} = \kappa^n$ is the marginal rate of substitution between good $n$ and good 0. Accordingly, $\kappa$ governs the rate at which the agent's marginal valuation increases across higher goods.

Lifetime utility is given by:

$$\int_0^{\infty} e^{-\rho t} U dt, \tag{8}$$

where $\rho > 0$ represents the subjective discount rate, interpreted as the social rate of time preference.

For this economy, the initial capital stock $Z_0 > 0$, the initial pollution abatement technology level $M_0 > 0$, and the initial pollution stock $P_0 \geq 0$ are given.

Since our objective is to study the joint evolution of industrial upgrading and pollution dynamics, we restrict attention to an economy that has already entered the industrialization process. Accordingly, we assume that the initial capital endowment $Z_0$ is not too low. Otherwise, the working capital stock would remain zero for an initial period, during which only sector 0 would be active.

As the economy at $t = 0$ is assumed to be at an early stage of industrialization, the stock of the common pollutant is naturally very small at the outset. We therefore assume that the initial pollution stock $P_0$ is sufficiently small and, for simplicity, normalize it to $P_0 = 0$.

For convenience, let $\beta = 1 - \alpha$, $b = \kappa\varepsilon$, and $a_0 = 0$, $a_n = a^n$, for any $n \in \mathbb{N}\backslash\{0\}$. And denote $\mathscr{A} = \{2a_n\}_{n\in\mathbb{N}}$. As for the parameters, we impose the following

**Basic Assumption**. $\rho < \eta < \rho/\alpha, \quad d = a > b + 1.$

The economic meaning of this assumption is as follows.

The condition $\eta > \rho$ ensures that the regeneration rate of capital outpaces the social rate of time preference. Otherwise, capital accumulation would be insufficient to sustain economic development, leading to the collapse of the economy.

Condition $\rho > \alpha\eta$ requires that the social rate of time preference not be too small. If $\rho$ were lower than $\alpha\eta$, the economy would exhibit explosive growth, implying an infinite level of social welfare. In such a case, the notion of social optimality would lose its economic meaning.

Condition $b > 1$ implies that consumers have a stronger intrinsic preference for higher goods relative to lower goods, abstracting from the effect of pollution (that is, after netting out pollution damages).

The condition $a > b$ rules out the trivial case in which higher capital intensity always dominates in the social planner's view, so that no well-defined optimal organization of production exists across industries. The stronger restriction $a > b + 1$ further guarantees that the rental price of capital is decreasing in the aggregate capital stock, a property closely related to the fact that good 0 requires no capital.

Finally, the assumption $d = a$ is imposed purely for analytical convenience and is not essential. Under a positive Pigouvian tax on industry-specific pollution, this assumption implies that, for any polluting industry $n$, the ratio of capital input $K_n$ to abatement effort $A_n$ equals one. Relaxing this assumption would alter this ratio but would not qualitatively affect the main results of the paper.

The above economy features two distinct sources of externalities, both of which lead to inefficiencies under laissez-faire market allocation. The first is a negative externality generated by pollution, which includes both industry-specific, non-accumulative pollutants and a common, accumulative pollutant. The second is a positive externality associated with technological progress in the abatement of common pollution due to knowledge spillovers and non-excludability, as investment in abatement technology generates economy-wide benefits that cannot be fully internalized by individual market participants.

But these inefficiencies can be solved through policy-plus-market approach. More precisely, the social optimum can be decentralized as a competitive equilibrium through a suitable combination of Pigouvian taxes and non-distortionary public financing. The Pigouvian taxes internalize the negative externalities from pollution, while the technology fund contribution internalizes the positive externality from abatement-technology progress[4]. Under suitable policies, the market equilibrium allocation coincides with the socially optimal allocation.

Therefore, we first solve the social planner's problem and characterize the socially optimal path of economic development. We then discuss how this allocation can be decentralized through a policy-plus-market approach.

We present two types of results. One type is more technical in nature and is stated as lemmas; the other type consists of results with substantive economic implications and is stated as propositions.

[4]Note that in our setup, technological change occurs only in the field of common-pollution abatement. For simplicity, we do not adopt Romer's approach of introducing a separate R&D sector endowed with monopoly power. Instead, we assume that the government establishes a fund to finance technological improvements by levying a lump-sum charge on households.

# 5 Social Optimum

We now turn to the characterization of the social optimum, focusing on the joint evolution of industrial upgrading and pollution.

The social planner's objective is to maximize social welfare subject to technological feasibility. Formally, the social planner's problem ($\mathbb{P}^s$) is given by

$$\begin{aligned}
\max \quad & \int_0^\infty e^{-\rho t}\left[\left(L_0+\sum_{n=1}^{\infty} b^n \min\left\{\frac{K_n}{a^n}, L_n, \frac{A_n}{a^n}\right\}\right)^{\alpha} - \sigma P\right] dt, && (9)\\
\text{s.t.} \quad & \dot{Z}=\eta Z-K-A, \quad \dot{P}=MK-\delta P, \quad \dot{M}=-AM, && (10)\\
& \sum_{n=1}^{\infty}(K_n+A_n)=K, \quad \sum_{n=0}^{\infty} L_n=1, && (11)\\
& Z(0)=Z_0, \quad P(0)=0, \quad M(0)=M_0. && (12)
\end{aligned}$$

Problem ($\mathbb{P}^s$) can be solved in two steps. First, solve the static problem ($\mathbb{P}_1^s$):

$$\begin{aligned}
\pi(K):= \quad \max \quad & L_0+\sum_{n=1}^{\infty} b^n \min\left\{\frac{K_n}{a^n}, L_n, \frac{A_n}{a^n}\right\}, && (13)\\
\text{s.t.} \quad & \sum_{n=1}^{\infty}(K_n+A_n)=K, \quad \sum_{n=0}^{\infty} L_n=1, && (14)
\end{aligned}$$

where $K \geq 0$ is given and $\pi(K)$ denotes the associated value function.

Second, solve the dynamic problem ($\mathbb{P}_2^s$)

$$\begin{aligned}
\max \quad & \int_0^\infty e^{-\rho t}\left(\pi^{\alpha}(K)-\sigma P\right) dt, && (15)\\
\text{s.t.} \quad & \dot{Z}=\eta Z-K-A, \quad \dot{P}=MK-\delta P, \quad \dot{M}=-AM, && (16)\\
& Z(0)=Z_0, \quad P(0)=0, \quad M(0)=M_0. && (17)
\end{aligned}$$

Furthermore, problem ($\mathbb{P}_1^s$) is equivalent to problem ($\mathbb{P}_1$):

$$\begin{aligned}
\max \quad & \sum_{n=0}^{\infty} b^n L_n, && (18)\\
\text{s.t.} \quad & \sum_{n=0}^{\infty} a_n L_n = K/2, \quad \sum_{n=0}^{\infty} L_n=1, && (19)
\end{aligned}$$

where $K \geq 0$ is given.

## 5.1 Solution of the social planner's problem

We now present the key results of the solution of ($\mathbb{P}^s$).

### 5.1.1 Solution of ($\mathbb{P}_1$)

For ($\mathbb{P}_1$), we have the following result, proof of which can be found in Appendix.

**Lemma 1**. The unique solution of ($\mathbb{P}_1$) is as follows:

$$L_n = \frac{a_{n+1} - K/2}{a_{n+1} - a_n}, \quad L_{n+1} = \frac{K/2 - a_n}{a_{n+1} - a_n}, \tag{20}$$

and $L_{n'} = 0$ for all $n' \neq n, n+1$, where $n = \max\{i \in \mathbb{N} \big| a_i \leq K/2\}$.

Therefore,

$$\pi(K) = \sum_{n=0}^{\infty} \left(k_n(K/2 - a_n) + b^n\right) I\left(K \in [2a_n, 2a_{n+1})\right), \tag{21}$$

where $I(\cdot)$ is the indicator function, and

$$k_n = \frac{b^{n+1} - b^n}{a_{n+1} - a_n}, \quad n \in \mathbb{N}. \tag{22}$$

Obviously, $\{k_n\}_{n\in\mathbb{N}}$ is strictly decreasing and converges to 0, and $\pi$ is continuous, concave and piecewise linear.

From Lemma 1, it follows that at any stage of economic development, at most two adjacent industries are active, while all other industries exit. Among all industries, the capital intensities $a_n$ and $a_{n+1}$ of these active industries $n$ and $n+1$ are the closest to one half of the aggregate capital–labor ratio, $K/2$ (recall that aggregate labor is normalized to unity). As a result, the factor allocation at any given time is entirely determined by the contemporaneous level of working capital $K$.

In other words, at each point in time, the production structure of the economy is fully pinned down by the factor endowment structure, namely the level of $K$. Therefore, to characterize the development of this economy, it is crucial to study the properties of the time path of $K$, which can only be obtained by solving ($\mathbb{P}_2^s$).

### 5.1.2 Solution of $(\mathbb{P}_2^s)$

A complete solution is provided in the Appendix. Here we emphasize the main properties of the time paths of $K$, $MK$, $A$ and $P$.

**Lemma 2**. (i) The time path of $K$ is continuous and piecewise smooth, alternating between intervals of strict increase and intervals of constancy on $\mathscr{A}$, and satisfies $\lim_{t\to\infty} K(t) = \infty$. (ii) For any $t \geq 0$,

$$M(t)K(t) = \frac{(\rho+\delta)\eta m}{\sigma} e^{(\rho-\eta)t}. \tag{23}$$

(iii) For any $t \geq 0$ away from corner points of $K(t)$,

$$A(t) = \eta - \rho + \dot{K}(t)/K(t). \tag{24}$$

(iv) For any $t \geq 0$,

$$P(t) = \begin{cases} \frac{(\rho+\delta)\eta m}{\sigma(\eta-\rho-\delta)} \left(e^{-\delta t} - e^{-(\eta-\rho)t}\right), & \text{if} \quad \eta \neq \rho+\delta, \\ \frac{(\rho+\delta)\eta m}{\sigma} t e^{-\delta t}, & \text{if} \quad \eta = \rho+\delta. \end{cases} \tag{25}$$

The constant $m$ appearing in (23) and (25) is the same positive constant, representing the shadow price of capital stock at time $t = 0$.

Lemma 2 implies that the abatement-technology investment $A$ reveals a distinct "pulse-plateau" cycle that correlates perfectly with the change of working capital $K$. And, the time path of common pollution flow $MK$ is strictly decreasing and tends exponentially to 0 as $t \to \infty$; the time path of common pollution stock $P$ is inverted-U-shaped and tends to 0 as $t \to \infty$, the peak time of $P$ is

$$T = \begin{cases} \frac{\ln(\eta-\rho)-\ln\delta}{(\eta-\rho)-\delta}, & \text{if} \quad \eta \neq \rho+\delta, \\ \frac{1}{\delta}, & \text{if} \quad \eta = \rho+\delta. \end{cases} \tag{26}$$

## 5.2 Life cycle and industrial upgrading

By Lemma 1 and Lemma 2, we obtain the following result immediately.

**Proposition 1.** In general, each industry $n$ has a finite life cycle consisting of three phases: growth, peak, and decline, which correspond exactly to three states of the working capital $K$: $K \in (2a_{n-1}, 2a_n)$, $K = 2a_n$, and

$K \in (2a_n, 2a_{n+1})$. More precisely, the output $Y_n$, or, equivalently, the labor demand $L_n$, of industry $n$ depends on the working capital $K$ according to

$$Y_n = \phi_n(K) := \begin{cases} \frac{K/2 - a_{n-1}}{a_n - a_{n-1}}, & \text{if} \quad K \in (2a_{n-1}, 2a_n), \\ 1, & \text{if} \quad K = 2a_n, \\ \frac{a_{n+1} - K/2}{a_{n+1} - a_n}, & \text{if} \quad K \in (2a_n, 2a_{n+1}), \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

Consequently, the time path of output $Y_n$ over the life cycle of industry $n$ exhibits a trapezoidal pattern: it increases from 0 to 1, remains at 1 for a finite time interval, and then declines from 1 to 0.

**Remark 1.** Exceptions to this generic pattern may arise only at the initial stage of economic development, where one or two industries which may not experience their entire life cycles, but only part of them. For expositional simplicity, throughout the sequel, discussions of life cycles and related topics are understood to hold in general, thereby excluding these exceptional cases.

Concerning the durations of these phases, we have the following result.

**Proposition 2**. In general, for each industry $n$, the durations of its growth, peak, and decline phases are given by

$$\Delta_{n1} = \frac{1}{\eta - \rho} \left[ \beta \ln b + \ln \frac{2 + \eta a^{-n}}{2 + \eta a^{-(n-1)}} \right], \tag{28}$$

$$\Delta_{n2} = \frac{1}{\eta - \rho} \ln \frac{a}{b}, \tag{29}$$

$$\Delta_{n3} = \frac{1}{\eta - \rho} \left[ \beta \ln b + \ln \frac{2 + \eta a^{-(n+1)}}{2 + \eta a^{-n}} \right], \tag{30}$$

respectively, and the total duration of its life cycle is

$$\Delta_n = \frac{1}{\eta - \rho} \left[ \ln a + (2\beta - 1) \ln b + \ln \frac{2 + \eta a^{-(n+1)}}{2 + \eta a^{-(n-1)}} \right]. \tag{31}$$

We observe that the duration of the peak phase is constant across industries, while the durations of the growth and decline phases are increasing in $n$ and converge as the economy develops. Consequently, the total duration of an industry's life cycle converges to

$$\Delta = \frac{1}{\eta - \rho} \left[ \ln a + (2\beta - 1) \ln b \right]. \tag{32}$$

As for industrial upgrading, the following result is immediate.

**Proposition 3.** Over the course of economic development, industrial upgrading proceeds sequentially, driven by changes in the factor endowment structure. Moreover, the speed of industrial upgrading, measured by $1/\Delta_n$, converges to a constant $1/\Delta$. This limit speed is increasing in $(\eta - \rho)$, decreasing in $\beta$ and $a$, decreasing in $b$ when $\beta > 1/2$, increasing in $b$ when $\beta < 1/2$, and irrelevant to $b$ when $\beta = 1/2$.

The comparative statics admits a clear economic interpretation.

The parameter $(\eta - \rho)$ captures the effective strength of capital accumulation, reflecting either faster capital regeneration or greater patience; both forces facilitate earlier entry into more capital-intensive industries and thus accelerate industrial upgrading.

A higher coefficient of relative risk aversion $\beta$ strengthens households' desire for consumption smoothing across time and industries, making structural transitions more costly in utility terms and thereby slowing down the upgrading process.

The parameter $a$ measures the increase in capital requirements across industries; a larger $a$ implies that more capital is required to move to the next industry, which naturally reduces the speed of industrial upgrading.

The role of $b$ is more subtle and depends critically on households' intertemporal preferences. The parameter $b$ governs the relative utility weight attached to higher goods after abstracting from industry-specific pollution, and thus captures the demand-side incentive to shift consumption toward more advanced industries. An increase in $b$ raises the relative attractiveness of higher goods, strengthening the incentive for industrial upgrading.

When $\beta < 1/2$, households are relatively willing to substitute consumption across industries and over time, so a higher $b$ amplifies the utility gain from entering higher industries and accelerates industrial upgrading.

By contrast, when $\beta > 1/2$, households place greater emphasis on consumption smoothing; in this case, a higher $b$ magnifies the utility gap across industries and reinforces the incentive to remain longer in the current industry, thereby slowing down the upgrading process.

When $\beta = 1/2$, these two opposing effects exactly offset each other, rendering the limiting upgrading speed independent of $b$.

### 5.3 Pollution dynamics

For industry-specific pollution, in general, pollution generated by industry $n$ is given by

$$P_n = \theta^n (1 - \varepsilon^n) L_n. \tag{33}$$

By (27), this implies

$$P_n(t) = \theta^n (1 - \varepsilon^n) \phi_n(K(t)), \quad \forall t \geq 0. \tag{34}$$

We therefore obtain the following result.

**Proposition 4.** In general, for each polluting industry, the time path of industry-specific pollution exhibits a trapezoidal pattern consistent with its life cycle.

Turning to common pollution, Lemma 2 yields the following characterization.

**Proposition 5.** Over the course of economic development, the time path of aggregate common pollution follows an inverted-U-shaped pattern, featuring a unique peak and eventual convergence to zero. Moreover, the peak time $T$ is strictly decreasing in $\eta$ and $\delta$ but strictly increasing in $\rho$, and satisfies

$$\lim_{\eta \to \infty} T = 0, \quad \lim_{\eta \to \rho} T = \infty. \tag{35}$$

Unlike industry-specific pollution, which is tightly linked to sectoral life cycles, common pollution reflects long-run accumulation and abatement dynamics. Its evolution is driven by the interaction between capital regeneration, endogenous improvements in abatement technology, and the natural decay of the pollution stock.

The peak time $T$ of the common pollution stock admits a transparent economic interpretation. Along the optimal path, pollution dynamics are governed by a race between two forces: the inflow of pollution, captured by the term $M(t)K(t)$, and the natural decay of the pollution stock at rate $\delta$.

A higher regeneration capacity $\eta$ accelerates structural upgrading and capital deepening, which in turn drives the pollution inflow $M(t)K(t)$ down more rapidly over time. As a result, the point at which pollution inflow falls below natural decay is reached earlier, leading to a smaller peak time $T$. This explains why $T$ is strictly decreasing in $\eta$.

A higher natural decay rate $\delta$ strengthens the self-cleaning ability of the environment. For any given pollution inflow, a larger fraction of the existing pollution stock is dissipated each period, so the stock reaches its maximum earlier and declines faster thereafter. Consequently, $T$ is strictly decreasing in $\delta$.

By contrast, a higher discount rate $\rho$ reflects greater impatience and places less weight on future environmental damages. This weakens the planner's incentive to front-load abatement and structural adjustment, allowing pollution inflows to remain relatively high for longer. As a result, the pollution stock accumulates over a longer horizon before turning downward, implying that $T$ is strictly increasing in $\rho$.

These forces imply that the pollution peak occurs arbitrarily early when regeneration is sufficiently strong, $\lim_{\eta\to\infty} T = 0$, while it is postponed indefinitely when regeneration barely offsets discounting, $\lim_{\eta\to\rho} T = \infty$.

These results provide a unified account of pollution dynamics over the development process. The model generates a sequence of transitional EKCs at the industry level, alongside a single long-run EKC for common pollution. Importantly, both patterns arise endogenously from structural change and technological progress, rather than from exogenous shocks.

Finally, it should be emphasized that the EKCs identified here are defined along the time path of economic development, rather than with respect to contemporaneous income levels. In this sense, they are time-based, rather than income-based, EKCs. See Section 8 for further discussion.

## 5.4 Dynamics of abatement-technology investment and pollution intensity

The “pulse–plateau” cycle of abatement-technology investment $A$ is tightly linked to the dynamics of working capital $K$, a pattern that is illustrated more transparently in the numerical simulations reported in Section 9. Periods of rapid expansion in productive capacity call for temporarily higher abatement effort to offset the scale effect on pollution, whereas once expansion ceases, a constant maintenance level of abatement investment becomes optimal.

It is important to emphasize that our analysis characterizes a socially optimal development path, rather than a positive description of historical growth experiences. In the present framework, optimal policy prescribes environmental intervention already at the early stage of industrialization. Although productive capacity expands as capital accumulates, the socially optimal response is to intensify investment in pollution abatement technology so as to reduce emission intensity sufficiently fast. As a result, the decline in emission intensity dominates the scale effect of production, and the pollution flow $M(t)K(t)$ falls monotonically over time rather than exhibiting an inverted-U shape. The inverted-U-shaped EKC therefore arises at the level of pollution stocks rather than contemporaneous emission flows, reflecting accumulation dynamics rather than delayed policy responses.

The dynamics of abatement-technology investment make this mechanism transparent. Away from corner points, optimal investment satisfies $A = \eta - \rho + \dot{K}/K$, so that periods of rapid capital expansion are mechanically accompanied by elevated abatement effort. During plateau phases, when productive capacity remains constant and pollution characteristics are already well understood, abatement investment optimally reverts to a constant maintenance level. When new industries emerge, however, discrete increases in abatement investment are required to prevent pollution flows from surging at the outset.

Thus, the joint dynamics of abatement investment $A$ and emission intensity $M$ characterize a coherent socially optimal development path, in which industrial upgrading proceeds alongside proactive environmental control rather than being followed by it. The resulting EKC patterns should therefore be understood as outcomes of optimal intertemporal policy design, rather than as mere historical regularities.

# 6 Decentralized implementation

The social optimum can be implemented through a decentralized mechanism employing three policy instruments.

First, a lump-sum technology charge is levied on households to finance investment in pollution abatement technologies. Second, a tax on capital income is introduced to align private capital returns with their socially optimal counterparts. Third, Pigouvian taxes on firms—or, alternatively, a system of tradable permits—are used to internalize pollution externalities.

All tax revenues are rebated to households in a lump-sum manner. Throughout most of this section, we focus on Pigouvian taxes and defer the discussion of tradable permits to the end of the section.

To avoid confusion and for convenience, we explicitly denote the solution of problem ($\mathbb{P}^s$) as $(Z^*, P^*, M^*, K^*, A^*, K_n^*, A_n^*, L_0^*, L_n^*, C_0^*, C_n^*, P_n^*, n \geq 1)$, and the shadow prices of capital stock $Z$, common pollution $P$ and emission intensity $M$ as $\lambda_Z^*$, $-\lambda_P^*$ and $-\lambda_M^*$, respectively. And, for any $t \geq 0$, let $X^*(t) = (C_0^*(t), C_n^*(t), P^*(t), P_n^*(t), n \geq 1)$.

Superscript $^*$ indicates variables evaluated at the solution of the social planner's problem ($\mathbb{P}^s$).

## 6.1 Policies

Three policy instruments are introduced.

**Technology fund**. To internalize the positive externality associated with technological progress in common-pollution abatement, the government levies a lump-sum charge on households at each point in time $t \geq 0$, denoted by $\chi(t)$.

The proceeds are used to finance investment in abatement technology. Since the charge is lump-sum, it does not affect households' marginal decisions. To replicate the planner's allocation, the technology fund contribution is set equal to the socially optimal investment path:

$$\chi(t) = A^*(t), \qquad \forall t \geq 0. \tag{36}$$

**Capital income tax.** At each time $t \geq 0$, a capital income tax is imposed at the rate

$$\xi(t) = \frac{1}{1 + \gamma K^*(t)/\eta}, \tag{37}$$

where $\gamma \in [\varepsilon, 1)$ is a constant. The parameter $\gamma$ can be interpreted as a capital income tax adjustment coefficient.

The role of the capital income tax is not fiscal or redistributive. Rather, it serves to address a structural distortion in capital returns and to facilitate the decentralized implementation of the socially optimal allocation. This distortion arises because the scarcity of working capital and the shadow value of the capital stock are jointly relevant for the social valuation of capital, whereas the market rental rate does not automatically reflect this composite scarcity. As a result, decentralized capital returns generally fail to replicate the shadow return characterized in the planner's problem.

The tax rate is therefore not chosen arbitrarily. First, the tax operates as a wedge rather than as a free policy instrument. It is designed so that the after-tax rental price of capital coincides with the scarcity rent (shadow value) of the capital stock measured in units of the numéraire, namely $\lambda_Z/U_{C_0}$. In this sense, the tax rate is endogenously determined by the implementation requirement, rather than selected in an ad hoc manner.

Second, the tax system makes explicit the scarcity of working capital. The term $\left(1 + \frac{\gamma K^*}{\eta}\right)$ effectively transforms the scarcity of working capital into a price wedge and embeds it in the market rental price of capital, so that the rental price takes the form $\frac{\lambda_Z}{U_{C_0}}\left(1 + \frac{\eta}{\gamma K^*}\right)$. The term $\left(1 + \frac{\eta}{\gamma K^*}\right)$ can be viewed as the scarcity rent associated with working capital. While this scarcity rent is implicit in the planner's problem, it is not automatically reflected in market prices.

The capital income tax renders this scarcity rent explicit in the decentralized economy, without administratively assigning rents.

Third, the parameter $\gamma$ does not represent the level of taxation per se, but rather governs the strength with which the scarcity of working capital is transmitted to market capital returns. A larger value of $\gamma$ corresponds to a lower tax rate. The lower bound $\gamma \geq \varepsilon$ ensures the feasibility of the construction of the Pigouvian tax below (see Lemma 4), while the restriction $\gamma < 1$ guarantees that the rental price of capital is strictly decreasing as capital accumulates.

**Pigouvian taxes**. Since two types of pollution are present, two corresponding Pigouvian taxes are imposed. For each industry $n \geq 1$ and at each time $t \geq 0$, an industry-specific emissions tax $\tau_n^s(t)$ is levied on net emissions of the $n$-th specific pollutant $P_n$, and a common-pollutant emissions tax $\tau_n^c(t)$ is levied on emissions contributing to the accumulation of the common pollution stock.

For sector $n$, emissions of the common pollutant take the form $E_n = M(K_n + A_n)$, where $K_n$ denotes working capital used in production, $A_n$ denotes investment in sector-specific pollution abatement, and $M$ is the emission intensity of common pollution. This formulation reflects the fact that both production activities and abatement efforts themselves generate common pollution.[5] All revenues from pollution taxes are rebated to households in a lump-sum manner, so that the tax system does not create additional income distortions.

## 6.2 Equilibrium

Let good-0 be the numéraire, with its price normalized to $p_0 = 1$. Although capital in the present model is not a natural resource in the literal sense, it is treated as a renewable input that is fully exhausted in production, in the same way as a raw material, and its rental price is denoted as $r$, the wage rate is denoted by $\omega$, and the price of good $n$ by $p_n$ for $n \geq 1$.

Given policies $(\chi, \xi, \tau_n^c, \tau_n^s, n \geq 1)$, where $\chi$ satisfies (36). An equilibrium is

[5]For example, installing and operating end-of-pipe abatement equipment—such as scrubbers or wastewater treatment facilities—requires additional capital and energy inputs, which generate carbon emissions and thereby contribute to the accumulation of common pollution.

a price system $(r, \omega, p_n, n \geq 1)$ and an allocation with which each agent (firm or agent) optimizes taking this price system as given, and all markets clear.

**Firms**. At each time $t$, firms in sector 0 choose labor $L_0$ to maximize instantaneous profit $L_0 - \omega L_0$. For each $n \geq 1$, firms in sector $n$ choose capital $K_n$, labor $L_n$, and abatement investment $A_n$ to solve

$$\begin{aligned} \max \quad & p_n Y_n - r(K_n + A_n) - \omega L_n - \tau_n^s P_n - \tau_n^c M(K_n + A_n), && (38) \\ \text{s.t.} \quad & P_n = \theta^n Y_n - Q_n, && (39) \\ & Y_n = \min\left\{\frac{K_n}{a^n}, L_n\right\}, && (40) \\ & Q_n = (\theta\varepsilon)^n \min\left\{Y_n, \frac{A_n}{a^n}\right\}. && (41) \end{aligned}$$

**Households**. Households supply one unit of labor and working capital $K$, pay the lump-sum technology charge $A^*$, receive factor income $\omega + (1-\xi)rK$ and lump-sum rebates $B$ from all taxes, and choose consumption $(C_n)_{n\geq 0}$ to maximize lifetime utility. That is, the problem of households is problem $(\mathbb{P}^h)$:

$$\begin{aligned} \max \quad & \int_0^\infty e^{-\rho t}\left[\left(C_0 + \sum_{n=1}^\infty \kappa^n \left(C_n - \theta^{-n} P_n\right)\right)^\alpha - \sigma P\right] dt && (42) \\ \text{s.t.} \quad & \dot{Z} = \eta Z - K - A^*, && (43) \\ & \sum_{n=0}^\infty p_n C_n = \omega + (1-\xi)rK + B, && (44) \\ & Z(0) = Z_0, && (45) \end{aligned}$$

where all prices, tax rates, pollution paths, lump-sum rebates $B$, and the technology investment path $A^*$ are taken as given. In particular, the fact that $(P, P_n, n \geq 1)$ is taken as exogenous by individual agents precisely captures the essence of the externality.

**Market clearing and pollution dynamics**. All markets clear. The evolution of common pollution satisfies

$$\dot{P} = M^* K - \delta P, \qquad P(0) = P_0, \tag{46}$$

where the emission intensity $M^*$ follows

$$\dot{M}^* = -A^* M^*, \qquad M^*(0) = M_0. \tag{47}$$

Total lump-sum rebates satisfy

$$B = \sum_{n=1}^{\infty} (\tau_n^c M^*(K_n + A_n) + \tau_n^s P_n) + \xi r K. \tag{48}$$

### 6.3 Pigouvian Tax system

An appropriate Pigouvian tax system decentralizing the social optimum is not unique. In what follows, we present one explicit construction to illustrate the mechanism.

Our basic idea is to ensure that, at each point in time, from the demand side, only the socially optimal adjacent goods, good $n$ and good $n+1$, are demanded, the prices of which are exactly equal to $\kappa^n$ and $\kappa^{n+1}$, respectively, and the price of each good $n' \neq 0, n, n+1$ is strictly greater than $\kappa^{n'}$; from the supply side, for each sector, the good price is exactly equal to its unit-cost.

A component of the household's problem can be equivalently written as

$$\max \quad \sum_{n=0}^{\infty} \kappa^n C_n \tag{49}$$

$$\text{s.t.} \quad \sum_{n=0}^{\infty} p_n C_n = J, \tag{50}$$

where $p_n$ denotes the price of good $n$ and $J$ is total income. Under this formulation, if prices satisfy the property that, when $K^* \in (2a_n, 2a_{n+1})$, we have that $p_n = \kappa^n$, $p_{n+1} = \kappa^{n+1}$, and $p_{n'} > \kappa^{n'}$ for all other $n' \neq 0$, then only sectors $n$, $n+1$ and sector 0 can possibly be active in equilibrium, while all other sectors necessarily receive zero demand and hence cannot be activated.

We then turn to the supply side and show how such a price configuration can be implemented through an appropriate tax system.

**Design of the tax system.** First, define

$$r^* := \frac{\lambda_Z^*}{U_{C_0}\big|_{X^*}} \left(1 + \frac{\eta}{\gamma K^*}\right). \tag{51}$$

Economically, $r^*$ represents the shadow value of capital, measured in units of the numéraire and adjusted by a factor inversely related to the level of working capital. We construct a tax system under which the equilibrium rental price of capital coincides with $r^*$. This rental price reflects the joint scarcity of two capital-related objects in the economy. The first is the capital stock $Z$, whose scarcity is captured by its shadow price $\lambda_Z^*$. The second is working capital $K$, whose scarcity is embodied in the term $\left(1+\frac{\eta}{\gamma K^*}\right)$, which can be interpreted as a "scarcity rent" associated with the availability of productive capital.

Now, for any $n \geq 1$ and any $t \geq 0$, we choose nonnegative taxes $(\tau_n^c(t), \tau_n^s(t))$ such that

$$p_n^*(t) \begin{cases} = \kappa^n, & \text{if} \quad K^*(t) \in (2a_{n-1}, 2a_{n+1}), \\ > \kappa^n, & \text{otherwise}, \end{cases} \tag{52}$$

where

$$p_n^* := 2a^n\big(r^* + \tau_n^c M^*\big) + 1 + \tau_n^s \theta^n (1-\varepsilon^n), \quad \forall n \geq 1. \tag{53}$$

This $p_n^*$ corresponds to the unit cost of producing good $n$ in sector $n$, evaluated at the optimal emission intensity $M^*$, the prevailing factor prices (the rental rate of capital $r^*$ and the wage rate $\omega = 1$), and the tax system $(\tau_n^c, \tau_n^s)$.

Is such a construction of $(\tau_n^c(t), \tau_n^s(t))$ feasible? Clearly, the central requirement of the above design is that, whenever $K^* \in (2a_{n-1}, 2a_{n+1})$, the unit cost of producing good $n$ must exactly equal $\kappa^n$:

$$2a^n\big(r^* + \tau_n^c M^*\big) + 1 + \tau_n^s \theta^n (1-\varepsilon^n) = \kappa^n. \tag{54}$$

As to the case, where the working capital lies outside this interval, there is sufficient flexibility to choose strictly positive taxes $(\tau_n^c, \tau_n^s)$ such that

$$2a^n\big(r^* + \tau_n^c M^*\big) + 1 + \tau_n^s \theta^n (1-\varepsilon^n) > \kappa^n, \tag{55}$$

thereby preventing good $n$ from demanding.

The feasibility of this construction hinges on basic properties of $r^*$. We state them as two lemmas.

**Lemma 3**. The time path of $r^*$ is continuous and strictly decreasing.

**Lemma 4**. For any $t \geq 0$, if $K^*(t) \in (2a_{n-1}, 2a_{n+1})$ for some $n \geq 1$, then

$$2a^n r^*(t) + 1 < \kappa^n. \tag{56}$$

Obviously, Lemma 4 ensures that whenever sector $n$ is socially active, its unit production cost net of pollution taxes is strictly below $\kappa^n$. Consequently, suitable positive taxes $(\tau_n^c, \tau_n^s)$ can always be chosen to satisfy (54), so that sector $n$ remains active and its product is demanded.

We now state the main decentralization result.

**Proposition 6**. Under the policy constructed above, there exists a competitive equilibrium whose allocation replicates the social optimum. The associated price system is given by the wage rate $\omega = 1$, the rental price of capital $r^*$, and the price of good $n$ equal to $p_n^*$ for all $n \geq 1$.

It is worth noting that the equilibrium is not unique. There exist multiple competitive equilibria, only some of which replicate the social optimum. In general, replication of the social optimum is not guaranteed for all equilibria.

In particular, the following modification of the above equilibrium also constitutes a competitive equilibrium and replicates the social optimum: the allocation, the wage rate, and the rental price of capital remain unchanged, while the price of good $n \geq 1$ is modified to $p_n(t)$ such that $p_n(t) = p_n^*(t)$ if $K^*(t) \in (2a_{n-1}, 2a_{n+1})$, and $p_n^*(t) > p_n(t) > \kappa^n$ otherwise.

To see this, note that when $K^*(t) \in (2a_{n-1}, 2a_{n+1})$, the equilibrium coincides with the original one. When $K^*(t) \notin (2a_{n-1}, 2a_{n+1})$, the condition $p_n(t) > \kappa^n$ ensures that good $n$ is not demanded by households, while the condition $p_n^*(t) > p_n(t)$ guarantees that good $n$ is not produced by firms. Consequently, allocations remain unchanged, and the overall economic outcome is unaffected.

This gives rise to an indeterminacy issue: under a purely Pigouvian approach, not only the price system but also the equilibrium allocation may fail to be unique[6].

Our primary focus is on the uniqueness of the equilibrium allocation and its exact coincidence with the socially optimal allocation. Even when the equilibrium allocation is unique, the associated price systems need not be, and such

[6]See also Anderson and Duanmu (2025) for a concrete example of multiple equilibria under Pigouvian taxation.

price indeterminacy may matter for price-based measures such as GDP.

As discussed in Section 6.5 below, a Coasean approach can be employed to guarantee the uniqueness of the equilibrium allocation.

At the end of this section, we provide an interpretation of the monotonicity of the rental price of capital $r^*$ established in Lemma 3.

Along a growth segment, both capital stock and working capital increase over time. The expansion of capital stock reduces its shadow value, while the growth of working capital weakens its "scarcity rent". As both sources of scarcity are gradually relaxed, the return to capital declines.

Along a plateau, working capital remains fixed and its "scarcity rent" is therefore constant. However, capital stock continues to accumulate in the background, so that its shadow value keeps falling. As a result, even in the absence of further expansion in working capital, the rental price of capital continues to decline.

Hence, the monotonic decline of $r^*$ does not hinge on whether the economy is in a growth phase or on a plateau, but instead reflects a persistent easing of capital scarcity as the economy progresses through successive stages of industrial upgrading.

## 6.4 Dynamic adjustment of taxes

The tax system constructed in the previous section is sufficient to decentralize the social optimum. However, it does not yet clarify how the taxes $(\tau_n^c, \tau_n^s)$ should be chosen so as to reflect economically meaningful measures of environmental damages, nor how they should evolve over time.

We now return to the tax system $(\tau_n^c, \tau_n^s, n \geq 1)$ and study its dynamic adjustment.

At the early stage of economic development, capital is relatively scarce, the unit-cost conditions derived above tightly constrain the policy space, leaving little scope for adjustment. As a result, they do not yet deliver a sharp or informative prescription for the choice of $\tau_n^c$ or $\tau_n^s$.

As time passes, capital accumulates and the level of working capital increases accordingly. Once capital has grown sufficiently large so that $K > 2a_n$ for some large $n$, a meaningful degree of policy flexibility emerges: it becomes possible to adjust either $\tau_n^c$ or $\tau_n^s$ so as to satisfy condition (54).

We now examine this point in more detail. Define

$$\tau := \frac{\lambda_P^*}{\lambda_Z^*}, \quad \tau_n := -\frac{U_{P_n}}{U_{C_0}}\bigg|_{X^*}. \tag{57}$$

The economic interpretations of $\tau$ and $\tau_n$ are as follows. The tax rate $\tau$ can be viewed as the shadow price of a claim on future marginal environmental damages generated by one additional unit of current emissions of common pollution, measured in units of capital. It therefore reflects an intertemporal, forward-looking valuation. In contrast, the tax rate $\tau_n$ is static and captures the marginal welfare damage generated by one additional unit of current emissions of industry-specific pollution, measured in units of the numéraire. We refer to $\tau$ and $\tau_n$ as the *standard* Pigouvian tax rates for common pollution and industry-specific pollution, respectively.

Concerning the magnitude of $\tau$ and $\tau_n$, we have the following result.

**Lemma 5**. If $K^* > 2a^n$ for some sufficiently large $n$, then

$$2a^n(r^* + \tau M^*) + 1 + \tau_n\theta^n(1 - \varepsilon^n) < \kappa^n. \tag{58}$$

Hence, if one were to simultaneously set

$$\tau_n^c = \tau, \quad \tau_n^s = \tau_n, \tag{59}$$

condition (55) would fail to hold.

This observation implies that, although both $\tau$ and $\tau_n$ possess clear and economically meaningful interpretations as Pigouvian tax rates, their direct combination is insufficient to implement the social optimum. The reason is that these *standard* Pigouvian taxes are designed to internalize marginal environmental damages, but they do not, by themselves, provide sufficiently strong price signals to coordinate sectoral activity and enforce the industry-selection structure.

From an implementation perspective, the tax policy $(\tau, \tau_n, n \geq 1)$ is therefore too weak: at least one of the two tax components must be strengthened so as to generate prices that are fully consistent with the optimal industrial composition.

Therefore, we adopt at most one of the two *standard* rates, either $\tau_n^c = \tau$ or $\tau_n^s = \tau_n$, and adjust the other so as to ensure that condition (54) remains satisfied.

In practice, we prefer to fix $\tau_n^c = \tau$ and adjust $\tau_n^s$ accordingly. The underlying rationale is twofold.

First, $\tau_n$ affects the economy primarily through short-run, sector-specific production decisions and can therefore be adjusted flexibly without altering long-run environmental dynamics.

Second, $\tau$ governs the intertemporal valuation of common pollution and plays the role of a long-run environmental shadow price; frequent or discretionary adjustments of $\tau$ would undermine its dynamic consistency. For these reasons, it is natural to anchor $\tau_n^c$ at the standard level $\tau$ and use $\tau_n^s$ as the residual instrument to achieve decentralization.

More generally, this policy design reflects a separation principle. The common-pollution tax $\tau$ is anchored at its standard Pigouvian level and serves as a long-run, dynamically consistent shadow price governing intertemporal environmental trade-offs, while the sector-specific taxes $\tau_n^s$ are adjusted flexibly to coordinate sectoral activity at the extensive margin and enforce the optimal production structure at each point in time.

Finally, regarding the dynamic properties of the standard tax rate $\tau$, we have the following result.

**Proposition 7**. The dynamics of $\tau$ satisfy

$$\dot{\tau} = (\eta + \delta)\tau - \upsilon, \tag{60}$$

where

$$\upsilon = -\frac{U_P\big|_{X^*}}{\lambda_Z^*}. \tag{61}$$

This equation admits a clear intertemporal interpretation. From a social perspective, $\tau$ can be viewed as an investment in the mitigation of common pollution. Such an investment requires a rate of return equal to $\eta + \delta$, which compensates both for the natural regeneration of capital and for the natural decay of the pollution stock. The instantaneous dividend of this investment is precisely the current marginal environmental damage $\upsilon$.

By the definition of $\tau$ and the properties of $\lambda_P^*$ and $\lambda_Z^*$ (see Appendix I), we have

$$\lim_{t\to\infty} e^{-(\eta+\delta)t}\tau(t) = 0, \tag{62}$$

it follows that, for any $t \geq 0$,

$$\tau(t) = \int_t^{\infty} e^{-(\eta+\delta)(s-t)}\upsilon(s)\, ds, \tag{63}$$

which shows that the tax $\tau(t)$ exactly capitalizes the future stream of marginal environmental damages from time $t$ onward.

## 6.5 Coasean approach

In principle, the indeterminacy discussed above can be resolved through a Coasean arrangement that assigns and trades pollution rights in a way that uniquely determines the allocation, even though the supporting equilibrium prices may remain indeterminate. In what follows, we construct such a Coasean mechanism and show that it decentralizes the social optimum exactly.

The government issues two types of tradable pollution permits: permits for common pollution and permits for industry-specific pollution. All permits are initially allocated to the representative agent. Firms must purchase the required permits in competitive permit markets in order to operate. Each type of permit constitutes a separate competitive market.

At any point in time $t \geq 0$, the total quota of permits for common pollution is given by

$$E(t) = M^*(t)K^*(t), \tag{64}$$

while the total quota of industry-specific pollution permits for industry $n \geq 1$ is given by

$$E_n(t) = P_n^*(t)\, I\big(t \in (T'_{n-1}, T_{n+1})\big), \tag{65}$$

where $(T'_{n-1}, T_{n+1})$ is the life-cycle interval of industry $n$, during which the socially optimal working capital satisfies $K^*(t) \in (2a_{n-1}, 2a_{n+1})$.

The technology fund contribution and the capital income tax are arranged in the same manner as above. An equilibrium under the Coasean arrangement can be defined analogously to that under Pigouvian taxation.

It is straightforward to verify that such an equilibrium exists. While the associated price system may not be unique, the resulting allocation is uniquely determined and coincides exactly with the socially optimal allocation.

One equilibrium price system supporting the social optimum is characterized as follows. The price of good 0 is normalized to $p_0 = 1$, the wage rate is equal to 1, the rental price of capital is $r^*$ in (51), and the price of good $n$ is $p_n^*(t)$. The price of a permit for common pollution is $\tau$ in (57), and the price of a permit for industry-specific pollution in industry $n$ is $\tau_n^s$ which satisfies the following conditions: if $t \in (T'_{n-1}, T_{n+1})$, that is, if industry $n$ is active, then,

$$\kappa^n = p_n^*(t) = 2a_n\big(r^*(t) + \tau(t)M^*(t)\big) + 1 + \tau_n^s(t)\theta^n(1 - \varepsilon^n); \tag{66}$$

if $t \notin (T'_{n-1}, T_{n+1})$, then

$$\kappa^n < p_n^*(t) = 2a_n\big(r^*(t) + \tau(t)M^*(t)\big) + 1 + \tau_n^s(t)\theta^n(1 - \varepsilon^n). \tag{67}$$

When $t \notin (T'_{n-1}, T_{n+1})$, industry $n$ is inactive and the quota of industry-specific pollution permits is 0. In this case, the corresponding permit market does not operate. One may equivalently imagine that such a market exists but that the permit price $\tau_n^s(t)$ is sufficiently high to ensure that the unit cost of production exceeds the output price, thereby preventing operation of the industry.

This price system coincides with the one obtained under the Pigouvian tax regime discussed in Section 6.4: the price of each pollution permit is equal to the corresponding Pigouvian tax rate.

Due to the Leontief nature of the production technology, all inputs and pollution emissions are required in fixed proportions to output. Firms therefore choose output levels solely based on the availability of pollution permits. The permit constraints bind exactly at the socially optimal quantities, ensuring that production, capital usage, and pollution emissions coincide with the social optimum.

In summary, although equilibrium prices under the Coasean arrangement may remain indeterminate as in the Pigouvian approach, the resulting allocation is unique. The Coasean mechanism therefore acts as an explicit equilibrium selection device, eliminating allocative multiplicity.

# 7 Macroeconomic dynamics

To study macroeconomic dynamics, we focus on GDP as the central aggregate variable, defined as

$$G := \sum_{n=0}^{\infty} p_n Y_n. \tag{68}$$

Since the equilibrium price system is not unique—under either the Pigouvian tax regime or the Coasean arrangement—GDP is, in principle, not uniquely defined. Throughout this section, we therefore evaluate GDP under a particular supporting price system, namely the one characterized in Section 6.4, in which $p_n = \kappa^n$ when sector $n$ is active. This price system supports the socially optimal allocation.

In equilibrium, at any point in time at most two adjacent industries can be active, depending on the level of working capital $K$. As a result, GDP depends only on $K$.

More precisely, for any $n \geq 0$, when $K \in (2a_n, 2a_{n+1}]$, we have $Y_{n'} = 0$ for all $n' \neq n, n+1$, with prices $p_n = \kappa^n$ and $p_{n+1} = \kappa^{n+1}$, and outputs given by (27). It follows that GDP can be expressed as a function of working capital:

$$G = \varphi(K), \tag{69}$$

where

$$\varphi(K) := \sum_{n=0}^{\infty} \frac{\kappa^n}{a_{n+1} - a_n} \left[ (a_{n+1} - \kappa a_n) + (\kappa - 1)\frac{K}{2} \right] I\big(K \in (2a_n, 2a_{n+1}]\big), \tag{70}$$

where $I(\cdot)$ denotes the indicator function.

The function $\varphi$ is continuous, strictly increasing, piecewise linear, and concave. Moreover,

$$\lim_{K\to\infty} \varphi(K) = \infty, \qquad \lim_{K\to\infty,\, K\notin\mathscr{A}} \varphi'(K) = 0, \tag{71}$$

and $\mathscr{A}$ is exactly the set of corner points of $\varphi$.

Combining this result with Lemma 1 and Lemma 2 yields the following characterization of aggregate output dynamics.

**Proposition 9.** The time path of GDP is continuous and piecewise smooth, alternating between intervals of strict increase and intervals of constancy, and diverges to infinity as time tends to infinity.

This alternating pattern arises from the interaction between the continuous evolution of working capital and the endogenous life-cycle dynamics of industries.

Periods of constancy correspond to *industry dominance regimes*, in which a single industry operates at its peak scale while all other industries have exited. In such regimes, although the dominant industry may be technologically mature and productive, the absence of sectoral coexistence eliminates further margins for aggregate expansion, leading to temporary stagnation of GDP.

By contrast, periods of strict increase correspond to *coexistence regimes*, in which two adjacent industries are simultaneously active. During these phases, emerging industries expand while incumbent ones have not yet fully declined. This overlap creates additional margins for resource reallocation and output expansion, resulting in strictly increasing aggregate output.

Taken together, aggregate growth does not follow a smooth exponential trajectory. Instead, it emerges from the structural dynamics of industrial upgrading, characterized by the rise, coexistence, and eventual decline of successive industries.

**Proposition 10.** Economic development exhibits endogenous growth–stagnation cycles driven by structural transitions between phases of industry dominance and industry coexistence, even in the absence of exogenous shocks.

These growth–stagnation cycles at the aggregate level are the macroeconomic counterpart of the trapezoidal output paths characterizing individual industries. Aggregate fluctuations therefore arise endogenously from the sequencing and overlap of sectoral life cycles, rather than from external disturbances.

This mechanism highlights the role of structural change as a fundamental source of medium-term macroeconomic dynamics, complementing growth frameworks that emphasize endogenous industrial evolution.[7]

# 8 EKC: time-based versus income-based

Throughout the paper, EKCs are defined along the time path of economic development and reflect the joint evolution of capital accumulation, structural change, and pollution abatement technologies. The analysis therefore focuses on *time-based* EKCs, as opposed to the conventional *income-based* EKCs that relate pollution to contemporaneous income or output.

This distinction is crucial. In the present framework, income alone is not a sufficient state variable to characterize pollution dynamics, especially in the presence of accumulative pollutants and endogenous technological progress. As a result, the non-monotonic pollution patterns derived below should not be interpreted as reduced-form income–pollution relationships, but as outcomes of an underlying dynamic process driven by industrial upgrading and long-run abatement dynamics.

[7] In Ju, Lin, and Wang (2015), each industry features only two phases—growth and decline—without a peak phase. As a consequence, aggregate output grows monotonically over time and does not exhibit growth–stagnation cycles. From a mathematical perspective, this difference can be traced to the fact that the continuity of the working capital path is not explicitly exploited in their analysis.

### 8.1 Industry-specific pollution

By (69), we know that GDP $G = \varphi(K)$, where $\varphi$ is strictly increasing, which implies $K = \varphi^{-1}(G)$. By (34), for any $n \geq 1$, $P_n = \theta^n(1-\varepsilon^n)\phi_n(K)$. Therefore,

$$P_n = \theta^n(1 - \varepsilon^n)\phi_n\left(\varphi^{-1}(G)\right). \tag{72}$$

That is, industry-specific pollution can be expressed as a function of GDP. Since $\phi_n \circ \varphi^{-1}$ is also inverted-U-shaped with a unique peak, then, as GDP rises, industry-specific pollution first increases and then decreases, giving rise to a classical EKC in the $(G, P_n)$ space. This reflects the essentially static and short-run nature of industry-specific pollution, which does not exert persistent effects on the long-run development path.

In addition, as capital accumulates and industrial upgrading proceeds sequentially, such EKCs emerge and disappear one after another across industries. These EKCs are therefore transitional in nature.

We have the following result, which complements and contrasts with Proposition 4.

**Proposition 4'.** In general, for each polluting industry, industry-specific pollution follows an inverted-U-shaped life-cycle pattern as GDP per capita increases. Across industries, economic development gives rise to a sequence of transitional EKCs, as incumbent industries decline and new industries emerge.

### 8.2 Common pollution

The situation is fundamentally different for common pollution. Even during stagnation phases, when GDP remains constant due to industry dominance, investment in abatement technology continues to reduce emission intensity $M$. As a result, common pollution keeps evolving over time even when GDP does not change.

Therefore, aggregate common pollution cannot be written as a function of GDP alone. In this sense, a classical income-based EKC does not exist for common pollution.

This observation cautions against interpreting empirical income–pollution relationships for accumulative pollutants as genuine EKCs, as income alone is not a sufficient state variable for their dynamics.

Instead, the relevant EKC for common pollution is a time-based one, emerging from the dynamic interaction between capital accumulation, abatement-technology progress, and natural decay.

# 9 Numerical simulation

We now conduct numerical simulations to illustrate the development path of the economy, focusing on the time paths of working capital $K$, sectoral outputs $\{Y_n\}_{n\geq 1}$, GDP $G$, common pollution stock $P$, emission intensity $M$, and abatement-technology investment $A$. The simulation starts from given initial conditions $(Z_0, M_0, P_0)$, where $P_0 = 0$.

## 9.1 Methodology

The main computational difficulty lies in the fact that the working capital path $K(t)$ is characterized by an implicit functional equation and depends on the unknown initial shadow price $m := \lambda_Z(0)$. We follow exactly the characterization derived in the Appendix I.

**Step 1 (the auxiliary function $\psi$).** For each $q > 0$, define $\psi(q)$ as the unique maximizer of the static problem (81). Lemma A provides a closed-form expression for $\psi$, given in (82)–(83), in terms of the threshold sequence $\{\lambda_n^+, \lambda_n^-\}$. The function $\psi$ is continuous, piecewise smooth, and decreasing on $(0, \infty)$, with $\lim_{q\to 0} \psi(q) = \infty$, while $\psi(q) = 0$ for $q \geq \lambda_0^+$.

**Step 2 (implicit characterization of $K(t)$).** Given $m > 0$, the socially optimal working capital path satisfies (111). Since $\psi$ is monotone and piecewise smooth, the equation in (111) can be solved numerically on a time grid by fixed-point iteration at each $t$, using the previously computed value of $K$ as the initial guess. This procedure yields a continuous path with flat segments at

$K(t) = 2a^n$, consistent with Lemma 2.

**Step 3 (investment and identification of $m$).** Whenever $K(t) \notin \mathscr{A} := \{2a_n : n \in \mathbb{N}\}$, optimal investment $A$ satisfies $A(t) = \eta - \rho + \dot{K}(t)/K(t)$ as in (110). In particular, on plateau segments where $K(t) \in \mathscr{A}$, we have $A(t) = \eta - \rho$. The initial capital stock $Z_0$ pins down $m$ through the integral identity given in (114). Numerically, for a candidate $m$, we first compute $K(t)$ and $A(t)$ as above, then evaluate the truncated integral on a sufficiently long horizon, and determine $m$ by a one-dimensional root search.

**Step 4 (pollution, sectoral output, and GDP).** Given $M_0$ and the optimal working capital path, the emission intensity $M(t)$ satisfies equation (23). The common pollution stock $P(t)$ admits a closed-form solution given in (25) (Lemma 2). Sectoral outputs are determined by the equilibrium allocation rules in Lemma 1: at each $t$, at most two adjacent industries are active, and their outputs depend only on $K(t)$. GDP is evaluated under the supporting price system $p_n = \kappa^n$ for active industries discussed in Section 6.4.

## 9.2 Parameter choice

The parameters are chosen to satisfy the Basic Assumption $\rho < \eta < \rho/\alpha$, $b > 1$, and $a > b + 1$. The initial conditions are $(Z_0, M_0, P_0)$ with $P_0 = 0$:

$$\rho = 0.03, \quad \alpha = 0.6, \quad \beta = 0.4, \quad \eta = 0.045, \quad \delta = 0.10, \quad \sigma = 1,$$

$$a = 3, \quad b = 1.8, \quad \kappa = 2, \quad \xi = 0.9, \quad Z_0 = 235, \quad M_0 = 0.6, \quad P_0 = 0.$$

Solving (114) together with (110) and (111) yields the initial shadow price and working capital:

$$m = \lambda_Z(0) \approx 0.07849068, \qquad K(0) = 6, \qquad T \approx 22.32.$$

Under these conditions, the economy starts from the peak phase of the first active industry ($K(0) = 2a$).

## 9.3 Simulation results

Figure 1 presents the simulation results for the real economy variables, illustrating the structural transformation process, explores the dynamics of environmental technology variables, highlighting the interaction between structural change and pollution control.

**Working capital** $K(t)$**.** As shown in Panel (a), the path is continuous and piecewise smooth, exhibiting a sequence of plateaus at $K(t) = 2a^n$, separated by strictly increasing segments. This "staircase" pattern confirms that capital accumulation is not uniform but punctuated by periods of structural consolidation (plateaus) and rapid expansion (transitions).

**Sectoral Outputs.** Panel (b) displays the life cycles of four successive industries ($Y_1$ to $Y_4$). Each industry follows a trapezoidal pattern: entry, peak (dominance), and decline. The overlap between adjacent industries ($Y_n$ and $Y_{n+1}$) during the transition phases is clearly visible, driving the aggregate growth dynamics. Since, for any active sector $n \geq 1$, industrial specific pollution satisfies $P_n = \theta^n(1 - \varepsilon^{)}Y_n$, it follows the same life-cycle pattern as sectoral output, differing only by a constant proportional factor. For this reason, industry-specific pollution is not plotted separately.

**GDP** $G(t)$**.** Panel (c) shows that GDP alternates between intervals of constancy and strict increase. The constant phases correspond to industry dominance regimes (where $K$ is on a plateau), while the growth phases correspond to coexistence regimes. This endogenous growth–stagnation cycle is a macroeconomic manifestation of the underlying discrete industrial upgrading process.

**Common pollution stock** $P(t)$**.** Panel (d) shows that $P(t)$ follows a unique inverted-U-shaped path. This confirms the existence of a time-based EKC for accumulative pollutants, driven by the interplay between pollution accumulation and abatement.

**Emission Intensity** $M(t)$**.** As shown in Panel (e), emission intensity decreases monotonically over time, driven by continuous investment in abatement technology. Notably, the rate of decline accelerates during transition phases (marked by vertical dashed lines), reflecting the intensified abatement efforts during periods of rapid industrial upgrading.

**Abatement-technology Investment** $A(t)$**.** Panel (f) reveals a distinct "pulse-plateau" cycle in abatement investment. During stagnation phases (plateaus), $A(t)$ remains at a baseline level $\eta - \rho$ (indicated by the horizontal gray line). This represents a "maintenance investment" where society pays the minimum environmental depreciation cost to maintain system balance in a mature industry. However, during transition phases (growth), $A(t)$ spikes significantly. This behavior highlights a dynamic complementarity between capital accumulation and pollution control: rapid industrial upgrading requires a simultaneous "big push" in clean technology to prevent environmental degradation from outpacing growth.

Figure 2 illustrates the mechanism underlying the unimodal time path of the common pollutant. At each point in time, the evolution of the pollution stock $P(t)$ is governed by the balance between the inflow $M(t)K(t)$ and the natural decay term $\delta P(t)$. When $M(t)K(t) > \delta P(t)$, pollution accumulates and $P(t)$ increases; when $M(t)K(t) < \delta P(t)$, pollution declines. The unique intersection of these two curves therefore identifies the turning point at which pollution reaches its peak.

## 9.4 Comparative statics of common pollution

This subsection examines the comparative statics of the common, accumulative pollutant with respect to three key parameters: the natural decay rate of pollution $\delta$, the discount rate $\rho$, and the environmental regeneration strength $\eta$. In all cases, the remaining parameters are held fixed, and the parameter restrictions stated in the **Basic Assumption** in Section 4 are satisfied, in particular $\rho < \eta < \rho/\alpha$.

Figure 3(a) illustrates the comparative statics with respect to the natural decay rate $\delta$. An increase in $\delta$ shifts the entire pollution path downward, reduces the peak level of pollution, and brings the peak forward in time. Economically,

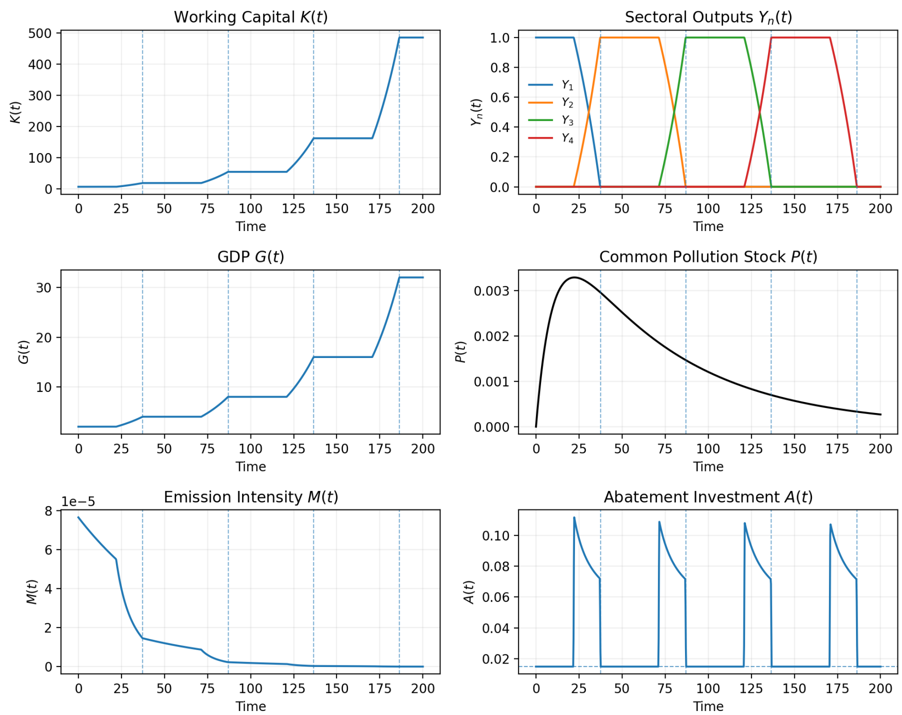



Figure 1: Time paths of working capital, sectoral outputs, GDP, common pollution, emission intensity, and abatement investment.

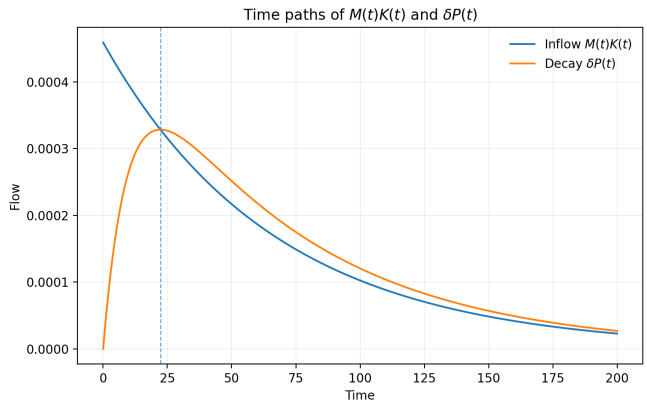



Figure 2: Time paths of the common-pollution inflow $M(t)K(t)$ and the decay term $\delta P(t)$. Their intersection identifies the peak time $T$ of $P(t)$.

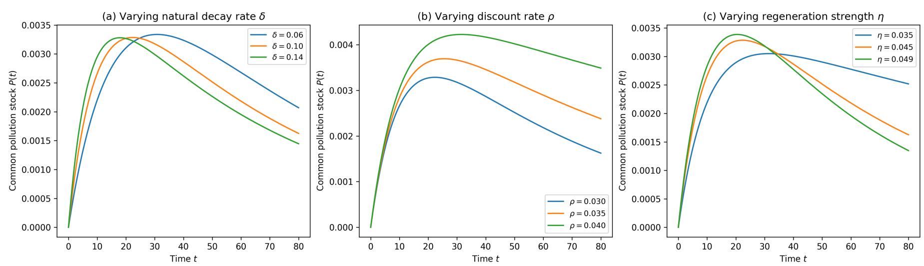



Figure 3: Time paths of common pollution under different natural decay rates $\delta$, discount rate $\rho$ and regeneration strength $\eta$.

a higher decay rate enhances the environment's ability to absorb and dissipate pollution, thereby mitigating accumulation while preserving the unimodal, inverted-U-shaped path of common pollution.

Figure 3(b) reports the comparative statics with respect to the discount rate $\rho$. A higher $\rho$ increases the weight placed on current production relative to future environmental quality, leading to a higher pollution peak and a more persistent accumulation of common pollution. Nevertheless, the qualitative inverted-U shape of the pollution path remains unchanged.

Figure 3(c) presents the comparative statics with respect to the regeneration strength $\eta$. An increase in $\eta$ strengthens the environment's capacity to recover from pollution, resulting in a lower peak level of common pollution and a faster transition toward cleaner states. In contrast to changes in the discount rate, a higher regeneration capacity mitigates pollution accumulation without relying on intertemporal trade-offs in preferences.

In summary, these figures show that while the timing and magnitude of common pollution accumulation are sensitive to economic preferences and environmental parameters, the inverted-U-shaped trajectory of common pollution is robust. These comparative statics highlight the distinct channels through which environmental persistence, intertemporal preferences, and regenerative capacity jointly shape pollution dynamics over the development process.

As for the comparative statics of the common, accumulative pollutant with respect to the marginal disutility parameter $\sigma$, equation (25) implies that $P(t)$ is strictly decreasing in $\sigma$ for every $t$. Economically, a higher $\sigma$ means that households place a larger marginal weight on common pollution in utility, which induces stronger incentives to mitigate it and therefore lowers the pollution stock at each point in time. At the same time, $\sigma$ affects $P(t)$ only through a multiplicative scaling factor in (25). Hence it shifts the entire path of $P(t)$ downward without changing its shape, and in particular it does not change the peak time. Because this effect is analytically transparent and involves only a proportional rescaling of the pollution path, an additional figure would not provide further economic insight and is therefore omitted.

# 10 Counterfactual analysis

This section studies how exogenous deviations in policy instruments reshape the market development path. Relative to the baseline economy, we perturb one instrument at a time and trace the induced changes in prices, industry turnover, capital dynamics, and pollution outcomes. These counterfactuals are designed to isolate the transmission mechanism of each instrument in a dynamic multi-industry environment.

The experiments fall into two categories. Counterfactuals I–II involve regime changes in policy design. Shutting down common-abatement progress ($A \equiv 0$) permanently alters incentives and leads to a new market equilibrium. In contrast, changing the tax adjustment parameter $\gamma$ modifies the rental price of capital while leaving the equilibrium allocation unchanged. Counterfactuals III–IV instead model external policy shocks: industry-specific and common-pollution tax rates are perturbed over a finite time window, after which the policy reverts to its baseline level and the market endogenously returns to the original equilibrium trajectory. Throughout, we compare the counterfactual paths to the baseline in terms of industry life cycles, the rental price of capital, and the time profile and composition of pollution.

## 10.1 Counterfactual I: $A \equiv 0$

We first consider a counterfactual in which investment in common-pollution abatement technology is exogenously shut down, so that $A \equiv 0$, while all other policy instruments remain unchanged.

With $A \equiv 0$, emission intensity remains constant at $M \equiv M_0$, and the law of motion becomes $\dot{P} = M_0 K - \delta P$. Since capital continues to accumulate under the baseline fiscal structure, the inflow term $M_0 K$ eventually dominates natural decay. Along the equilibrium path, $K(t) \to \infty$ and $P(t) \to \infty$, so pollution diverges monotonically.

Figure **??** contrasts the baseline inverted-U path with the counterfactual trajectory.

This experiment shows that abatement progress is essential for the existence of an aggregate EKC. Without abatement technology improvement, the common-pollution stock increases monotonically and diverges.

## 10.2 Counterfactual II: varying $\gamma$

We now vary the capital-income tax parameter $\gamma$. In plateau phases the rental price is $r = \frac{\lambda_Z}{U_{C_0}}\left(1+\frac{\eta}{\gamma K}\right)$, while in expansion phases it is $r = \pi'(K)\frac{K+\eta/\gamma}{K+\eta}$. In both expressions, $r$ is strictly decreasing in $\gamma$ pointwise in time. Therefore,

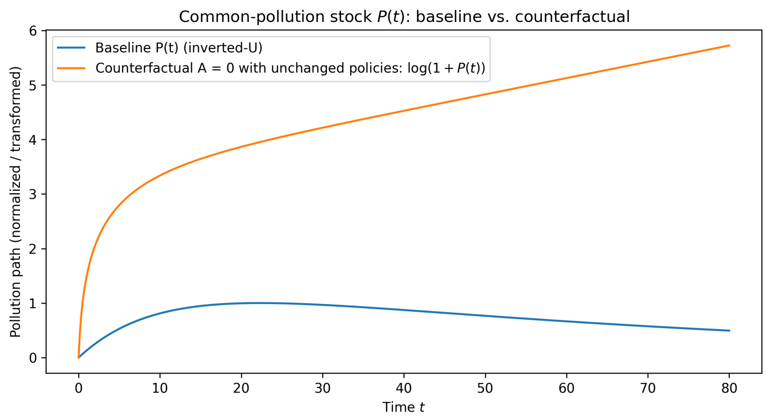
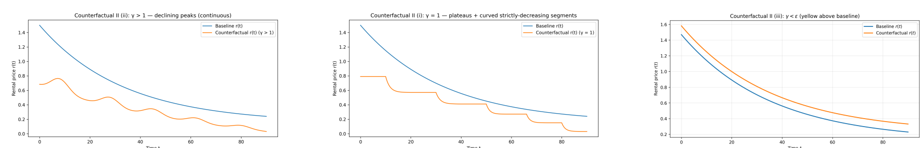



Figure 4: Counterfactual I: shutting down abatement progress ($A \equiv 0$).

an increase in $\gamma$ shifts the entire time path of $r$ downward, while a decrease shifts it upward. In particular, the resulting path is strictly ordered relative to the baseline and does not intersect it at any point in time.

Figure 5 plots rental-price paths under different values of $\gamma$.




(a) $\gamma > 1$ (b) $\gamma = 1$ (c) $\gamma < \varepsilon$

Figure 5: Effect of varying $\gamma$.

Changes in $\gamma$ reshape the price system but leave the socially optimal allocation unchanged. When $\gamma$ is sufficiently large, in particular when $\gamma > 1$, the monotonicity of $r(t)$ is reversed in expansion phases: despite the continuous increase in capital, the rental price $r(t)$ becomes strictly increasing. By contrast, in stagnation phases, $r(t)$ remains strictly decreasing. When $\gamma = 1$, the rental price remains constant during expansion phases, even though capital continues to increase, and remains strictly decreasing during stagnation phases. In both cases, government intervention is excessively strong, and the rental price fails to correctly reflect the scarcity of capital.

At the other extreme, if $\gamma$ becomes too small, specifically when $\gamma < \varepsilon$, implementing the socially optimal allocation may require negative Pigouvian taxes, which are economically implausible. Thus, implementability constraints impose meaningful bounds on admissible policy parameters beyond allocative efficiency considerations.

These observations imply that the economically meaningful range of $\gamma$ is given by $[\varepsilon, 1)$. Within this interval, government intervention is neither too weak nor too strong, and the price system preserves its economically interpretable role in reflecting capital scarcity while remaining consistent with implementability requirements.

## 10.3 Counterfactual III: varying $\tau_n^s$

We next examine a temporary increase in the industry-specific pollution tax $\tau_n^s$ for a currently active industry $n$.

The unit cost of industry $n$ is $c_n = 2a^n(r^* + \tau_n^c M^*) + 1 + \tau_n^s \theta^n (1 - \varepsilon^n)$. An increase in $\tau_n^s$ raises $c_n$. Because equilibrium requires $\kappa^n = c_n$, the rental price must adjust. Either $r$ decreases to maintain $\kappa^n = c_n$, implying $\kappa^{n+1} > c_{n+1}$ and immediate takeover by industry $n+1$, or $r$ increases to maintain $\kappa^{n+1} = c_{n+1}$, implying $\kappa^n < c_n$ and immediate exit of industry $n$. In both cases, structural upgrading is accelerated.

Figure 6 illustrates the shortened life cycle of industry $n$. Industry-specific taxation acts primarily as a timing instrument, influencing the speed of sectoral turnover without altering long-run structural direction.

## 10.4 Counterfactual IV: varying $\tau$

Suppose that the government adopts the standard Pigouvian tax on the common pollutant, with the tax rate $\tau$ defined in (57). We now consider a temporary external increase in this tax rate and examine how the resulting change in $\tau$ affects the market development path.

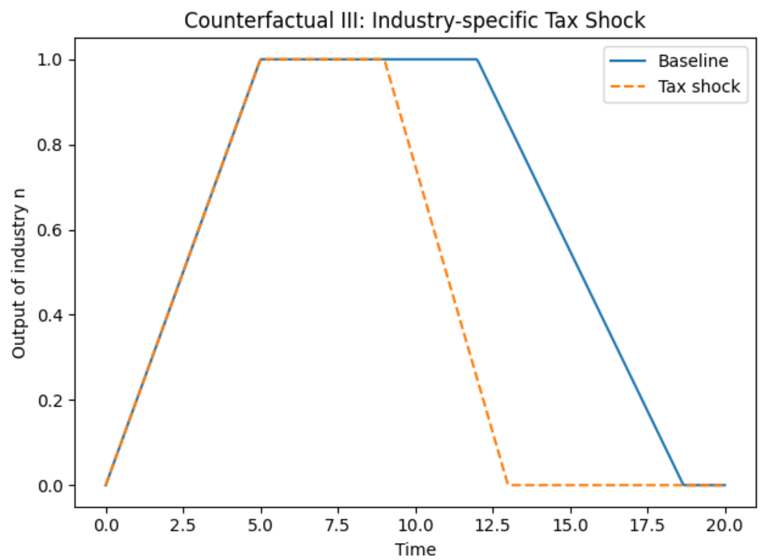



Figure 6: Counterfactual III: temporary increase in $\tau_n^s$.

Under this tax regime, the unit cost of industry $n$ becomes $c_n = 2a^n(r^* + \tau M^*) + 1 + \tau_n^s \theta^n(1 - \varepsilon^n)$. A temporary increase in $\tau$ raises production costs for all currently active industries so that $\kappa^n < c_n$ holds. As a consequence, currently active industries exit immediately and higher industries enter earlier, leading to an accelerated industrial upgrading.

Suppose that this policy intervention occurs at time $t_0 > 0$, prior to the baseline peak time $T$ of the common pollution path $P(t)$. The pollution stock at that moment is $P(t_0) > 0$. Since higher industries require more working capital $K$, while the aggregate capital stock $Z$ remains unchanged at that instant, capital stock becomes relatively more scarce. Its shadow price $\lambda_Z(t_0)$ therefore increases.

Referring to Lemma 2 (with the appropriate modification under the new initial condition), for $t \geq t_0$ the evolution of pollution satisfies

$$\dot{P}(t) = \frac{(\rho + \delta)\eta}{\sigma} \lambda_Z(t_0) e^{(\rho - \eta)(t - t_0)} - \delta P(t). \tag{73}$$

An increase in $\lambda_Z(t_0)$ therefore raises the pollution inflow term for all $t \geq t_0$, shifting the entire post-$t_0$ trajectory upward. Solving this linear differential equation yields the closed-form solution for $P(t)$ and, consequently, the peak time of the pollution path (assuming $\eta \neq \rho + \delta$):

$$T = t_0 + \frac{1}{\rho + \delta - \eta} \ln \left\{ \frac{\delta}{\eta - \rho} \left[ 1 + \frac{\sigma(\eta - \rho - \delta)}{\eta(\rho + \delta)} \frac{P(t_0)}{\lambda_Z(t_0)} \right] \right\}. \qquad (74)$$

Differentiating $T$ with respect to $\lambda_Z(t_0)$ shows that $T$ is strictly increasing in $\lambda_Z(t_0)$. Hence, a higher shadow price of capital at the moment of intervention not only raises the pollution level but also postpones the turning point of the inverted-U trajectory.

The case $\eta = \rho + \delta$ can be treated analogously, and the same monotonicity result holds.

Consequently, the common pollution path remains continuous at $t_0$, but for all $t > t_0$ it lies strictly above the baseline path. The trajectory continues to exhibit an inverted-U shape; however, both the peak level and the peak time increase. The peak time is therefore postponed relative to the baseline economy.

Figure 7 illustrates this mechanism. The temporary increase in $\tau$ accelerates industrial upgrading, but because higher industries are more capital-intensive, capital scarcity intensifies and the implied common pollution flow $MK$ rises. As a result, the pollution stock becomes larger than in the baseline case, and the inverted-U curve shifts upward with a delayed turning point.

At first glance, this result appears counterintuitive: raising the pollution tax leads to heavier pollution and a postponed peak. The mechanism, however, is internally consistent. The tax shock accelerates structural upgrading, but the higher capital intensity of advanced industries increases capital scarcity and raises its shadow value. Through the term $MK$, this amplifies the pollution inflow and shifts the entire pollution path upward.

Unlike industry-specific taxation, the common pollution tax operates as a macro-level structural lever. Even a temporary deviation reshapes intertemporal capital scarcity and alters the timing and magnitude of the aggregate pollution cycle.

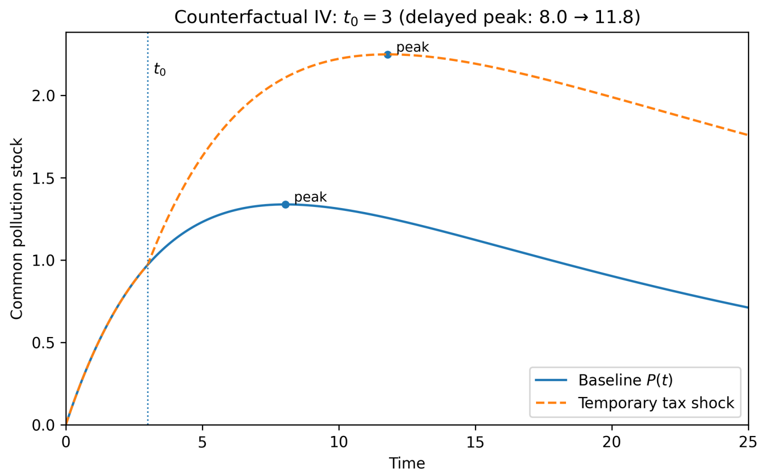



Figure 7: Counterfactual IV: temporary increase in $\tau$.

## Summary of counterfactual experiments

The four counterfactual experiments reveal several distinct mechanisms through which policy interventions interact with structural transformation and environmental dynamics.

Counterfactual I demonstrates that abatement progress is a structural condition for the existence of the inverted-U pattern. Once the improvement of common-pollution abatement is shut down, the stabilizing force disappears and pollution diverges. In this sense, technological progress in abatement determines whether sustainable development is feasible at all.

Counterfactual II shows that institutional parameters governing capital-income taxation reshape intertemporal price incentives. While the qualitative pattern of industrial upgrading may remain, the entire rental-price path and the speed of capital accumulation are altered. Thus, fiscal design affects the long-run development trajectory through price dynamics.

Counterfactual III highlights the timing channel of industry-specific taxation. A temporary increase in sectoral pollution taxes modifies relative production costs and accelerates industry turnover, thereby re-timing structural upgrading without fundamentally changing the underlying growth mechanism.

Counterfactual IV reveals a deeper macro-level transmission mechanism. A temporary increase in the common pollution tax accelerates industrial upgrading toward more capital-intensive sectors. Given the predetermined capital stock, this raises capital scarcity and its shadow price, which in turn amplifies the pollution inflow term. As a result, the common pollution path shifts upward and its peak is postponed. What initially appears counterintuitive follows from the endogenous interaction between capital scarcity and pollution dynamics.

To summarize, the results underscore that environmental policy operates through multiple layers: technological constraints, intertemporal price incentives, sectoral cost differentials, and capital-scarcity feedback effects. The impact of policy therefore depends not only on its environmental objective, but also on how it interacts with the structure of capital accumulation and industrial upgrading. These findings imply that effective policy requires a coordinated combination of instruments. Isolated adjustments to any single instrument may distort intertemporal price signals and lead to welfare losses.

## 11 Conclusion

This paper develops a dynamic multi-sector growth model with pollution to study the joint evolution of industrial upgrading and environmental quality. The analysis highlights that a multi-sector perspective is essential for understanding EKC dynamics when pollution is heterogeneous and its composition changes endogenously over the development process.

By modeling industries as having finite life cycles consisting of growth, peak, and decline, the framework captures a central feature of structural transformation that is largely absent from existing EKC theories. Embedding environmental dynamics into this structure reveals how sectoral turnover generates non-monotonic pollution paths at the industry level and how the sequencing and overlap of industries shape aggregate pollution dynamics. Because different stages of economic development are associated with different dominant industries, they are also associated with different dominant types of pollution. As industries emerge and exit, not only the level of pollution but also its composition evolves endogenously.

A central implication of the model is that EKC dynamics need not rely on exogenous technological progress or delayed policy intervention. Instead, they can arise naturally from structural transformation itself. The distinction between industry-specific, non-accumulative pollution and common, accumulative pollution provides a coherent explanation for why some pollutants follow transitional, sector-specific EKC patterns, while others exhibit a single inverted-U-shaped path at the aggregate level. Moreover, the internal dynamics of industrial upgrading may generate endogenous growth–stagnation cycles even in the absence of external shocks.

In addition to characterizing the social optimum, the paper discusses its decentralized implementation. We construct a set of three policy instruments—a technology fund charge, a capital income tax, and either Pigouvian taxes or Coasian permit quotas—that jointly support a competitive equilibrium replicating the socially optimal allocation. To our knowledge, this hybrid policy arrangement has not been previously studied in the EKC literature and provides a concrete illustration of how environmental regulation and capital taxation can be combined to address multiple distortions arising from structural transformation and pollution externalities.

The counterfactual analysis further clarifies the multiple channels through which policy interacts with structural transformation. Shutting down abatement progress eliminates the inverted-U mechanism altogether, underscoring its structural necessity. Excessively strong or excessively weak capital-income tax intervention destroys either the monotonicity of the rental price of capital or the nonnegativity of the Pigouvian tax. Temporary sector-specific tax shocks primarily re-time industry turnover. Most strikingly, a temporary increase in the common pollution tax may accelerate industrial upgrading toward more capital-intensive sectors but the aggregate pollution path can shift upward and its peak may be postponed. This seemingly counterintuitive result reflects the endogenous interaction between capital accumulation, structural change, and environmental dynamics.

Beyond offering a theoretical explanation for observed pollution trajectories, the framework underscores structural transformation as a fundamental force shaping not only economic development but also the evolving structure of environmental degradation. By explicitly linking industrial upgrading to pollution dynamics within a unified general-equilibrium setting, the analysis bridges the literatures on structural change and environmental economics and highlights the importance of capital-scarcity feedback mechanisms in environmental policy design.

The model also opens several directions for future research. Empirically, it suggests new approaches to measuring composition effects that explicitly account for industry entry, exit, and overlap, rather than relying on reduced-form sectoral shares. On the policy side, it provides a natural foundation for studying how environmental regulation should adapt to changing pollution structures across stages of development. More broadly, extensions incorporating international trade, cross-country heterogeneity, or endogenous technological change would offer a richer platform for analyzing the interaction between industrial upgrading, environmental sustainability, and long-run growth.

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# Appendix I. Proofs

## Proof of Lemma 1

Since the objective function and the constraints are all linear, and at any feasible point, the constraints are linearly independent, then, $\{L_n\}_{n=0,1,2,...}$ is optimal, if and only if there exist Lagrangian multipliers $\mu$, $\nu$, $\{\theta_n\}_{n=0,1,2,...}$ such that for any $n = 0, 1, 2, ...,$

$$b^n - \mu - \nu a_n + \theta_n = 0, \tag{75}$$

$$\theta_n \geq 0, \quad L_n \geq 0, \quad \theta_n L_n = 0, \tag{76}$$

Thus, if there exist $n_1 < n_2 < n_3$ such that $L_{n_1} > 0, L_{n_3} > 0$, then,

$$b^{n_1} = \mu + \nu a_{n_1}, \quad b^{n_2} \leq \mu + \nu a_{n_2}, \quad b^{n_3} = \mu + \nu a_{n_3}, \tag{77}$$

and hence,

$$\frac{b^{n_3} - b^{n_2}}{a_{n_3} - a_{n_2}} \geq \frac{b^{n_2} - b^{n_1}}{a_{n_2} - a_{n_1}}. \tag{78}$$

But this is impossible, since it's easy to verify that

$$\frac{b^{n_3} - b^{n_2}}{a_{n_3} - a_{n_2}} < \frac{b^{n_2} - b^{n_1}}{a_{n_2} - a_{n_1}}. \tag{79}$$

So, there exists a unique $n \in \mathbb{N}$ such that $L_n > 0$, $L_{n+1} \geq 0$, $L_i = 0, \forall i \neq n, n+1$. It follows that

$$L_n + L_{n+1} = 1, \quad a_n L_n + a_{n+1} L_{n+1} = K/2, \tag{80}$$

which yields (20). Therefore, $K \in [2a_n, 2a_{n+1})$, which determines the $n$ corresponding to $K$, that is, $n = \max\{i \in \mathbb{N} \big| a_i \leq K/2\}$. And this completes the proof of Lemma 1.

Before presenting the solution of $(\mathbb{P}_2^s)$, we give a preliminary result, stated as a lemma, the proof of which is elementary, hence, omitted.

**Lemma A**. For any $q > 0$, the optimization problem

$$\max_{x \geq 0} \{\pi^\alpha(x) - qx\} \tag{81}$$

admits a unique solution, denoted as $\psi(q)$, which has the following closed form representation:

$$\psi(q) = \begin{cases} 2\left(a_n + \frac{1}{k_n}\left[\left(\frac{\alpha k_n}{2q}\right)^{1/\beta} - b^n\right]\right), & \text{if} \quad q \in [\lambda_{n+1}^-, \lambda_n^+), \\ 2a_n, & \text{if} \quad q \in [\lambda_n^+, \lambda_n^-), \end{cases} \tag{82}$$

where $\lambda_0^- = \infty$, and for any $n \in \mathbb{N}$,

$$\lambda_n^+ = \frac{\alpha k_n}{2} b^{-n\beta}, \quad \lambda_{n+1}^- = \frac{\alpha k_n}{2} b^{-(n+1)\beta}. \tag{83}$$

Clearly, $\psi$ is continuous, piece-wise smooth, decreasing on $(0, \infty)$, and satisfying $\lim_{q \to 0} \psi(q) = \infty$ and $\psi(q) = 0$, $\forall q \geq \lambda_0^+ = \alpha(b-1)/(2a)$.

**Proof**. Let $f(x) := \pi^\alpha(x)$. Note that $f$ is strictly concave and differentiable at any $x \notin \mathscr{A}$. For any $x \in \mathscr{A}$, the right and left derivatives $f'_+(x)$ and $f'_-(x)$ exist and satisfy $f'_-(x) > f'_+(x)$. Moreover, $f'_-(0) = \alpha(b-1)/(2a)$. For convenience, we set $f'_-(0) = \infty$.

It is easy to see that if a real number $q \in [f'_+(x),\ f'_-(x)]$ for some $x \geq 0$, then this $x$ is the unique solution to problem (81) with this $q$. Now, let $\lambda_n^+ = f'_+(2a_n)$, $\forall n \geq 0$; $\lambda_n^- = f'_-(2a_n)$, $\forall n \geq 1$. Then the assertion follows easily. The proof is completed.

## Proof of Lemma 2

Let the current-value Hamiltonian be

$$H^s = \pi^\alpha(K) - \sigma P + \lambda_Z(\eta Z - K - A) - \lambda_P(MK - \delta P) + \lambda_M(AM). \quad (84)$$

By the Pontryagin maximum principle, the optimal $(Z, P, M, K, A)$ satisfies the following conditions: there exists absolutely continuous and piecewise smooth $(\lambda_Z, \lambda_P, \lambda_M)$ such that[8]

$$\dot{\lambda}_Z = \rho\lambda_Z - H^s_Z = (\rho - \eta)\lambda_Z, \quad (85)$$

$$\dot{\lambda}_P = \rho\lambda_P + H^s_P = (\rho + \delta)\lambda_P - \sigma, \quad (86)$$

$$\dot{\lambda}_M = \rho\lambda_M + H^s_M = (\rho + A)\lambda_M - \lambda_P K, \quad (87)$$

$$0 = H^s_A = -\lambda_Z + \lambda_M M, \quad (88)$$

$$K(t) \in \arg\max_{x\geq 0}\{\pi^\alpha(x) - \lambda(t)x\}, \quad (89)$$

where

$$\lambda(t) := \lambda_Z(t) + \lambda_P(t)M(t), \quad (90)$$

and three transversality conditions (TVCs):

$$\lim_{t\to\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0, \quad (91)$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_P(t)P(t) = 0, \quad (92)$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_M(t)M(t) = 0. \quad (93)$$

By the above Lemma A, (89) implies

$$K(t) = \psi(\lambda(t)). \quad (94)$$

Therefore, $K(t)$ is continuous and piecewise smooth.

We now prove that $K(t)$ is increasing and that $\lim_{t\to\infty} K(t) = \infty$. To this end, it suffices to show that $\lambda(t)$ is decreasing and that $\lim_{t\to\infty} \lambda(t) = 0$.

[8]Equation (87) holds for any $t$ away from possible corner points. It will be seen in the sequel that the corner points of $\lambda_M$ coincide with those of $K(t)$; see (107).

We first solve the costate equations. From (85), it follows that

$$\lambda_Z(t) = me^{(\rho-\eta)t}, \tag{95}$$

where $m > 0$ is a constant. Equation (86) yields

$$\lambda_P(t) = \frac{\sigma}{\rho+\delta} + ce^{(\rho+\delta)t}, \tag{96}$$

where $c \geq 0$ is a constant.

Next, note that $\dot{P} \geq -\delta P$ and that there exists some $t_0 > 0$ such that $P(t_0) > 0$. Hence, for any $t > t_0$,

$$P(t) \geq P(t_0)e^{-\delta t}. \tag{97}$$

Together with the transversality condition (92), this implies

$$\lim_{t\to\infty} e^{-(\rho+\delta)t}\lambda_P(t) = 0, \tag{98}$$

which in turn yields $c = 0$. Consequently,

$$\lambda_P \equiv \frac{\sigma}{\rho+\delta}. \tag{99}$$

We now turn to the behavior of $\lambda(t)$. Expression (95) implies that $\lambda_Z(t)$ is strictly decreasing and converges to zero as $t \to \infty$. Moreover, $M(t)$ is decreasing. It therefore follows that $\lambda(t)$ is strictly decreasing.

To establish $\lim_{t\to\infty} \lambda(t) = 0$, it suffices to show that

$$\lim_{t\to\infty} M(t) = 0. \tag{100}$$

Recall from (88) that

$$\lambda_Z = \lambda_M M. \tag{101}$$

Differentiating both sides and using (85), (87), and $\dot{M} = -AM$, we obtain

$$(\rho-\eta)\lambda_Z = \dot{\lambda}_Z = \dot{\lambda}_M M + \lambda_M \dot{M} \tag{102}$$

$$= ((\rho+A)\lambda_M - \lambda_P K)\, M - \lambda_M AM \tag{103}$$

$$= \rho\lambda_M M - \lambda_P MK \tag{104}$$

$$= \rho\lambda_Z - \lambda_P MK. \tag{105}$$

Rearranging terms gives

$$\lambda_P MK = \eta \lambda_Z, \tag{106}$$

which, together with (95) and (99) and that $K(t)$ is increasing, implies (100).

We now turn to common pollution. Relations (95), (99), and (106) imply (23). Substituting the expression for $MK$ in (23) into the pollution dynamics $\dot{P} = MK - \delta P$ and using the initial condition $P(0) = 0$, we obtain (25), from which it follows that the peak time $T$ satisfies (26). Then, (35) follows immediately.

Notice that, by (101) and (106),

$$\lambda_P K = \eta \lambda_M, \tag{107}$$

which, together with (87), yields that for any $t$ away from the corner points of $K(t)$,

$$\dot{\lambda}_M = (\rho - \eta + A)\lambda_M. \tag{108}$$

Once again, by (107), we obtain that for any $t$ away from the corner points of $K(t)$,

$$\frac{\dot{\lambda}_M(t)}{\lambda_M(t)} = \frac{\dot{K}(t)}{K(t)}, \tag{109}$$

therefore, for any $t$ away from the corner points of $K(t)$,

$$A(t) = \eta - \rho + \frac{\dot{K}(t)}{K(t)}. \tag{110}$$

Moreover, by (90), (94), (95) and (106), we have, for any $t \geq 0$,

$$K(t) = \psi\left(me^{(\rho-\eta)t}\left(1 + \frac{\eta}{K(t)}\right)\right). \tag{111}$$

In addition, relations (91) and (95) imply that

$$\lim_{t\to\infty} e^{-\eta t} Z(t) = 0. \tag{112}$$

Together with the capital accumulation equation $\dot{Z} = \eta Z - K - A$, this yields, for any $t \geq 0$,

$$Z(t) = \int_t^{\infty} e^{-\eta s}\big(K(s) + A(s)\big)\,ds. \tag{113}$$

This representation implies that, at any point in time, the capital stock exactly supports the entire future demand for working capital and investment in pollution-abatement technology from that time onward. In particular,

$$Z_0 = \int_0^\infty e^{-\eta t}(K(t) + A(t))dt. \tag{114}$$

It then follows that $m$ is uniquely determined by (110), (111), and (114). Consequently, the social planner's problem ($\mathbb{P}^s$) admits a unique solution. This completes the proof of Lemma 2.

## Proof of Proposition 2

By (89), for any $t$ such that $K(t) \notin \mathscr{A}$,

$$\alpha\left(\pi(K(t))\right)^{-\beta}\pi'(K(t)) = \lambda(t). \tag{115}$$

Substituting the expressions implied by (90), (95), and (106), we obtain, for any $t$ such that $K(t) \notin \mathscr{A}$,

$$\alpha\left(\pi(K(t))\right)^{-\beta}\pi'(K(t)) = me^{(\rho-\eta)t}\left(1 + \frac{\eta}{K(t)}\right). \tag{116}$$

Now suppose that, over some time interval, $K(t) \in (2a^{n-1}, 2a^n)$ for some $n \geq 2$. By (21), within this interval we have $\pi'(K(t)) = k_{n-1}/2$. Substituting this into (116) yields

$$\alpha\left(\pi(K(t))\right)^{-\beta}k_{n-1} = 2me^{(\rho-\eta)t}\left(1 + \frac{\eta}{K(t)}\right). \tag{117}$$

It follows that there exists a point in time at which $K$ first reaches $2a^n$, which we denote by $T_n$. The variable $K$ then remains at $2a^n$ for a positive length of time, though not indefinitely, before subsequently entering the interval $(2a^n, 2a^{n+1})$. Let $T_n'$ denote the last time at which $K$ stays at $2a^n$.

By the continuity of $K(t)$ and $\pi$, taking the left limit $t \to T_n-$ yields

$$\alpha\left(\pi(K(T_n))\right)^{-\beta}k_{n-1} = 2me^{(\rho-\eta)T_n}\left(1 + \frac{\eta}{K(T_n)}\right), \tag{118}$$

while taking the right limit $t \to T_n+$ gives

$$\alpha\left(\pi(K(T_n'))\right)^{-\beta}k_n = 2me^{(\rho-\eta)T_n'}\left(1 + \frac{\eta}{K(T_n')}\right). \tag{119}$$

Noting that $K(T_n) = K(T'_n) = 2a^n$ and $\pi(K(T_n)) = \pi(K(T'_n)) = b^n$, we obtain

$$\Delta_{n2} := T'_n - T_n = \frac{1}{\eta - \rho} \ln \frac{a}{b}. \tag{120}$$

That is, $K$ remains at $2a^n$ over the time interval $[T_n, T'_n]$.

Similarly, $K$ stays at $2a^{n-1}$ over the time interval $[T_{n-1}, T'_{n-1}]$; $K$ stays at $2a^{n+1}$ over the time interval $[T_{n+1}, T'_{n+1}]$. And, it holds that

$$\alpha \left(\pi(K(T_{n+1}))\right)^{-\beta} k_n = 2me^{(\rho-\eta)T_{n+1}} \left(1 + \frac{\eta}{K(T_{n+1})}\right), \tag{121}$$

$$\alpha \left(\pi(K(T'_{n-1}))\right)^{-\beta} k_{n-1} = 2me^{(\rho-\eta)T'_{n-1}} \left(1 + \frac{\eta}{K(T'_{n-1})}\right). \tag{122}$$

Noticing $K(T_{n+1}) = 2a^{n+1}$, $\pi(K(T_{n+1})) = b^{n+1}$, and $K(T'_{n-1}) = 2a^{n-1}$, $\pi(K(T'_{n-1})) = b^{n-1}$, we obtain

$$\Delta_{n3} := T_{n+1} - T'_n = \frac{1}{\eta - \rho} \left[\beta \ln b + \ln \frac{2 + \eta a^{-(n+1)}}{2 + \eta a^{-n}}\right], \tag{123}$$

$$\Delta_{n1} := T_n - T'_{n-1} = \frac{1}{\eta - \rho} \left[\beta \ln b + \ln \frac{2 + \eta a^{-n})}{2 + \eta a^{-(n-1)}}\right]. \tag{124}$$

The time intervals $[T'_{n-1}, T_n)$, $[T_n, T'_n]$, and $(T'_n, T_{n+1})$ correspond to the growth phase, peak phase, and decline phase of industry $n$, respectively. And $\Delta_{n1}$, $\Delta_{n2}$ and $\Delta_{n3}$ are their durations, respectively. Hence, the total duration of the life cycle of industry $n$ is given by

$$\Delta_n := \Delta_{n1} + \Delta_{n2} + \Delta_{n3} = \frac{1}{\eta - \rho} \left[\ln \frac{a}{b} + 2\beta \ln b + \ln \frac{2 + \eta a^{-(n+1)})}{2 + \eta a^{-(n-1)}}\right]. \tag{125}$$

The proof is completed.

## Proof of Lemma 3

The continuity of $r^*(t)$ follows directly from its definition in (51). We now turn to its monotonicity.

It suffices to show that for any $n \geq 1$, $r^*(t)$ is strictly decreasing on the interval $(T'_{n-1}, T'_n]$, during which the economy is in either the growth phase or the peak phase of industry $n$.

First, consider $t \in (T'_{n-1}, T_n)$. In this case, $K^*(t) \notin \mathscr{A}$, then[9],

$$U_{C_0}\Big|_{X^*(t)} \pi'\big(K^*(t)\big) = \lambda_Z^*(t)\left(1 + \frac{\eta}{K^*(t)}\right). \tag{126}$$

Comparing (126) with (51), together with the fact that $\pi'(K^*(t)) = k_{n-1}/2$ on the interval $(T'_{n-1}, T_n)$, we obtain

$$r^*(t) = \frac{k_{n-1}}{2} \frac{K^*(t) + \eta/\gamma}{K^*(t) + \eta}. \tag{127}$$

Since $K^*(t)$ is strictly increasing on $(T'_{n-1}, T_n)$, it follows that $r^*(t)$ is strictly decreasing on this interval.

Next, consider $t \in [T_n, T'_n]$. Along this interval, the working capital is constant at the plateau level $K^*(t) = 2a_n$, and we have

$$r^*(t) = \frac{b^{n\beta}}{\alpha}\left(1 + \frac{\eta}{2\gamma a_n}\right)\lambda_Z^*(t). \tag{128}$$

Since $\lambda_Z^*(t)$ is strictly decreasing over $[T_n, T'_n]$, so is $r^*(t)$.

Combining the two cases, we conclude that $r^*(t)$ is strictly decreasing on $(T'_{n-1}, T'_n]$ for each $n \geq 1$. This completes the proof.

## Proof of Lemma 4

By Lemma 3, we know that whenever $K^*(t) \in (2a_{n-1}, 2a_{n+1})$ for some $n \geq 1$, we have

$$r^*(t) \leq \frac{k_{n-1}}{2\gamma}. \tag{129}$$

Thus, it suffices to show that for any $n \geq 1$,

$$a^n k_{n-1} + \gamma < \gamma \kappa^n, \tag{130}$$

for which, in turn, it suffices to prove that for any $n \geq 1$,

$$a^n k_{n-1} + 1 \leq b^n. \tag{131}$$

In fact, if $n = 1$, then,

$$ak_0 + 1 = a \cdot \frac{b-1}{a} + 1 = b. \tag{132}$$

[9] In fact, this coincides with (116), since $U_{C_0}\Big|_{X^*} = \alpha\pi^{-\beta}(K^*)$.

If $n \geq 2$, then,

$$
\begin{aligned}
a^n k_{n-1} + 1 &= a^n \left(\frac{b}{a}\right)^{n-1} \frac{b-1}{a-1} + 1 && (133)\\
&= \frac{a}{a-1}\frac{b-1}{b} b^n + 1 && (134)\\
&< \frac{b+1}{b}\frac{b-1}{b} b^n + 1 = (b^2-1)b^{n-2} + 1 && (135)\\
&= b^n - b^{n-2} + 1 \leq b^n. && (136)
\end{aligned}
$$

And this yields (131). The proof is completed.

## Proof of Proposition 6

Noticing (53), each firm's profit is maximized under the production arrangement implied by the socially optimal allocation, and the resulting maximized profit is zero.

We now turn to the household problem, namely, problem ($\mathbb{P}^h$). It can be solved in two steps. We first solve problem ($\mathbb{P}^h_1$):

$$
\begin{aligned}
\phi(K) := \quad & \max_{(C_n, n\in\mathbb{N})} \quad \sum_{n=0}^{\infty} \kappa^n C_n && (137)\\
& \text{s.t.} \quad \sum_{n=0}^{\infty} p_n^* C_n = J, && (138)
\end{aligned}
$$

where $J = 1 + (1-\xi)r^* K + B$ is the total income, $K$ is taken as given, and $r^*$, $B$ and ($p_n^*$, $n \geq 0$) are also given, with $p_0^* = 1$. $\phi$ denotes the associated value function.

We then solve problem ($\mathbb{P}^h_2$):

$$
\begin{aligned}
\max_{(Z,K)} \quad & \int_0^{\infty} e^{-\rho t} \left(\phi(K) - \sum_{n=1}^{\infty} (\kappa/\theta)^n P_n\right)^{\alpha} dt, && (139)\\
\text{s.t.} \quad & \dot{Z} = \eta Z - K - A^*, && (140)\\
& Z(0) = Z_0, && (141)
\end{aligned}
$$

where $Z_0$, $A^*$ and $(P_n)_{n\geq 1}$ are given.

First, we solve $(\mathbb{P}_1^h)$. It's easy to see that at any stage of the economic development, if $K \in (2a_n, 2a_{n+1}]$ for some $n \in \mathbb{N}$, notice that for any $i \geq 1$,

$$p_i^* \begin{cases} = \kappa^i, & \text{if} \quad i = n, n+1, \\ > \kappa^i, & \text{if} \quad i \neq n, n+1, \end{cases} \tag{142}$$

therefore, $(C_i)_{i\in\mathbb{N}}$ belongs to the solution of $(\mathbb{P}_1^h)$ if and only if $C_i = 0$ for any $i \neq 0, n, n+1$, and

$$C_0 + p_n^* C_n + p_{n+1}^* C_{n+1} = 1 + r^* K + B, \quad \text{if} \quad n > 0, \tag{143}$$

or

$$p_n^* C_n + p_{n+1}^* C_{n+1} = 1 + r^* K + B, \quad \text{if} \quad n = 0. \tag{144}$$

One can verify directly that the socially optimal $(C_i^*)_{i\in\mathbb{N}}$ satisfies all these conditions. Therefore, $(C_i^*)_{i\in\mathbb{N}}$ is a solution to $(\mathbb{P}_1^h)$.

And, accordingly,

$$\phi(K) = 1 + (1-\xi) r^* K + B. \tag{145}$$

Therefore, $\phi'(K) = (1-\xi)r^*$.

Next, solve $(\mathbb{P}_2^h)$. Let the current-value Hamiltonian be

$$H = \left(\phi(K) - \sum_{t=1}^{\infty} (\kappa/\theta)^n P_n\right)^{\alpha} - \sigma P + \lambda_Z(\eta Z - K - A^*). \tag{146}$$

By the Pontryagin maximum principle, the optimal $(Z, K)$ satisfies the following conditions: there exists absolutely continuous and piecewise smooth $\lambda_Z$ such that

$$\dot{\lambda}_Z = \rho\lambda_Z - H_Z = (\rho - \eta)\lambda_Z, \tag{147}$$

$$\alpha \left(\phi(K) - \sum_{t=1}^{\infty} (\kappa/\theta)^n P_n\right)^{-\beta} (1-\xi) r^* = \lambda_Z, \tag{148}$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} \lambda_Z(t) Z(t) = 0. \tag{149}$$

Notice that

$$\alpha \left(\phi(K^*) - \sum_{t=1}^{\infty} (\kappa/\theta)^n P_n^*\right)^{-\beta} = U_{C_0}\Big|_{X^*}. \tag{150}$$

Thus, the socially optimal $(Z^*, K^*)$ and the corresponding shadow price of capital stock $\lambda_Z^*$ satisfies all the conditions (140), (141), (147), (148) and (149). Therefore, $(Z^*, K^*)$ is a solution to $(\mathbb{P}_2^h)$.

Of course, the resulting pair $(M, P)$ coincides exactly with the socially optimal pair $(M^*, P^*)$. This follows from the fact that, given $A^*$ and the initial condition $M(0) = M_0$, the law of motion $\dot{M} = -A^*M$ uniquely determines $M$, which therefore equals $M^*$. Likewise, given $K^*$ and the initial condition $P(0) = 0$, the equation $\dot{P} = M^*K^* - \delta P$ uniquely determines $P$, which therefore coincides with $P^*$.

In addition, the total lump-sum rebates satisfies

$$B = \sum_{n=1}^{\infty} (\tau_n^c M^*(K_n^* + A_n^*) + \tau_n^s P_n^*) + \xi r^* K^*. \tag{151}$$

Consequently, the price system $(r^*, 1, p_n^*, n \geq 1)$ and the socially optimal allocation $(Z^*, P^*, M^*, K^*, A^*, K_n^*, A_n^*, L_0^*, L_n^*, C_0^*, C_n^*, P_n^*, n \geq 1)$ constitute an equilibrium. This completes the proof.

## Proof of Lemma 5

It is straightforward to verify that

$$\tau = \frac{\eta}{M^*K^*}, \quad \tau_n = \left(\frac{\kappa}{\theta}\right)^n. \tag{152}$$

If $K^* > 2a^n$ for some sufficiently large $n$, then, using an argument similar to that for (127), together with the strict decrease of the time path of $r^*$, and invoking the definition of $k_n$ in (22) as well as the basic assumption that $a - 1 > b$, we have

$$\begin{aligned} r^* &< \frac{k_n}{2} \frac{2a^n + \eta/\gamma}{2a^n + \eta} && (153) \\ &< \frac{k_n}{2} \left(1 + \frac{\eta}{2\gamma a^n}\right) && (154) \\ &< \left(\frac{b}{a}\right)^n \frac{b-1}{2b} \left(1 + \frac{\eta}{2\gamma a^n}\right), && (155) \end{aligned}$$

which implies

$$2a^n(r^* + \tau M^*) + 1 + \tau_n \theta^n (1 - \varepsilon^n) \tag{156}$$

$$= 2a^n(r^* + \eta/K^*) + 1 + \kappa^n - b^n \tag{157}$$

$$< \left(b^n - b^{n-1}\right)\left(1 + \frac{\eta}{2\gamma a^n}\right) + \eta + 1 + \kappa^n - b^n \tag{158}$$

$$< \kappa^n - b^{n-1} + \frac{\eta}{2\gamma}\left(\frac{b}{a}\right)^n + \eta + 1 \tag{159}$$

$$< \kappa^n. \tag{160}$$

The proof is completed.

### Proof of Proposition 7

In fact,

$$\tau(t) = \frac{\lambda_P}{m} e^{(\eta-\rho)t}, \quad \upsilon(t) = \frac{\sigma}{m} e^{(\eta-\rho)t}, \quad \lambda_P \equiv \frac{\sigma}{\rho + \delta}. \tag{161}$$

Then, (60) can be verified directly. The proof is completed.

## Appendix II. Index

### II-1 Variables

$A$ Abatement-technology investment for common pollution, reducing the emission intensity $M$; first introduced on p.12.

$A_n$ Abatement effort undertaken by industry $n$ for industry-specific pollution; first introduced on p.12.

$a_n$ Capital-intensity requirement of industry $n$, defined by $a_0 = 0$ and $a_n = a^n$ for $n \geq 1$; first introduced on p.14.

$B$ Lump-sum rebates; first introduced on p.29.

$C_n$ Consumption of good $n \geq 0$; first introduced on p.13.

$E$ Total quota of permits for common pollution under the Coasean policy regime; first introduced on p.33.

$E_n$ Quota of industry-specific pollution permits allocated to industry $n$; first introduced on p.33.

$G$ Gross Domestic Product (GDP), defined as the value of aggregate production; first introduced on p.35.

$J$ Total income of the representative agent; first introduced on p.30.

$K$ Aggregate working capital; first introduced on p.12.

$K_n$ Capital input employed in industry $n \geq 1$; first introduced on p.11.

$k_n$ Marginal productivity index for the mixture of industries $n$ and $n+1$, defined as $k_n = 2\pi'(K)$ for $K \in (2a_n, 2a_{n+1})$, that is, twice the slope of $\pi(K)$ over the corresponding feasibility interval; first introduced on p.17.

$L_n$ Labor allocated to industry $n \geq 0$; first introduced on p.11.

$M$ Emission intensity of the common accumulative pollutant; first introduced on p.12.

$m$ Shadow value of capital stock at the initial date; first introduced on p.18.

$n$ Index of industries ordered by increasing capital intensity; first introduced on p.10.

$P$ Stock of common (accumulative) pollution; first introduced on p.12.

$P_n$ Flow of industry-specific (non-accumulative) pollution generated by industry $n$; first introduced on p.11.

$p_n$ Price of the good $n$ produced by industry $n$; first introduced on p.25.

$Q_n$ Effective pollution reduction in industry $n$; first introduced on p.12.

$r$ Rental price of capital; first introduced on p.25.

$T$ Time at which the common pollution stock $P(t)$ reaches its peak; first introduced on p.18.

$T_n$ Starting time of the peak phase of industry $n$; first introduced on p.54.

$T'_n$ Ending time of the peak phase of industry $n$; first introduced on p.54.

$Y_n$ Output of industry $n \geq 0$; first introduced on p.11.

$\Delta_{n1}$ Duration of the growth phase of industry $n$; first introduced on p.19.

$\Delta_{n2}$ Duration of the peak phase of industry $n$; first introduced on p.19.

$\Delta_{n3}$ Duration of the decline phase of industry $n$; first introduced on p.19.

$\Delta_n$ Total life-cycle duration of industry $n$; first introduced on p.20.

$\Delta$ Limit of industry life-cycle duration, defined as $\Delta = \lim_{n\to\infty} \Delta_n$; first introduced on p.20.

$\chi$ Lump-sum charge levied on households to finance investment in pollution-abatement technology; first introduced on p.25.

$\omega$ Wage rate; first introduced on p.25.

$\tau$ Standard Pigouvian tax rate on common pollution; first introduced on p.30.

$\tau_n$ Standard Pigouvian tax rate on industry-specific pollution; first introduced on p.30.

$\tau_n^c$ Pigouvian tax on common pollution applied to industry $n$; first introduced on p.25.

$\tau_n^s$ Pigouvian tax on industry-specific pollution applied to industry $n$; first introduced on p.25.

$\upsilon$ Instantaneous marginal environmental damage of common pollution; first introduced on p.32.

$\xi$ Capital income tax rate; first introduced on p.28.

$^*$ Superscript indicating variables evaluated at the solution of the social planner's problem ($\mathbb{P}^s$); first introduced on p.24.

## II-2 Functions

$I(\cdot)$ Indicator function; first introduced on p.17.

$U(\cdot)$ Instantaneous utility function; first introduced on p.13.

$\pi(K)$ Socially optimal composite consumption (net of industry specific pollution) as a function of working capital $K$; first introduced on p.17.

$\phi_n(K)$ Industry-$n$ output as a function of aggregate capital; first introduced on p.19.

$\phi(K)$ value function of individual's problem $(\mathbb{P}_1^h)$; first introduced on p.19.

$\varphi(K)$ GDP as a function of working capital; first introduced on p.35.

$\psi(q)$ Solution of optimization problem (78); first introduced on p.50.

## II-3 Sets

$\mathbb{N}$ Set of non-negative integers; first introduced on p.11.

$\mathscr{A}$ Set of capital thresholds triggering industry entry and exit, $\mathscr{A} = \{2a_n\}_{n\in\mathbb{N}}$; first introduced on p.14.

## II-4 Parameters

$Z_0$ Initial value of capital stock; first introduced on p.13.

$P_0$ Initial value of common pollution stock; first introduced on p.14.

$M_0$ Initial value of common pollution emission intensity; first introduced on p.14.

$a$ Base parameter governing the spacing of capital-intensity thresholds across industries, with $a > 1$; first introduced on p.11.

$b$ Composite preference parameter governing the relative demand for higher-index industries, with $b = \kappa\varepsilon$; first introduced on p.14.

$d$ Elasticity parameter in industry-specific abatement technology, with $d > 0$; first introduced on p.12.

$\alpha$ Curvature parameter in social welfare, with $\alpha \in (0, 1)$; first introduced on p.13.

$\beta$ Output elasticity in utility, defined by $\beta = 1 - \alpha$; first introduced on p.14.

$\gamma$ Adjustment coefficient in the capital income tax rate, with $\gamma \in [\varepsilon, 1)$; first introduced on p.28.

$\delta$ Natural decay rate of the common pollution stock $P$, with $\delta > 0$; first introduced on p.12.

$\delta_1$ Depreciation rates of working capital, with $\delta_1 \in (0, 1]$; first introduced on p.12.

$\delta_2$ Depreciation rates of abatement-technology investment, with $\delta_1 \in (0, 1]$; first introduced on p.12.

$\varepsilon$ Effectiveness parameter of industry-specific abatement, with $\varepsilon > 0$; first introduced on p.12.

$\eta$ Capital regeneration (growth) rate in the AK-type accumulation process; first introduced on p.12.

$\kappa$ Relative preference weight for higher-index goods, with $\kappa > 1$; first introduced on p.13.

$\rho$ Social discount rate, with $\rho > 0$; first introduced on p.13.

$\sigma$ Marginal social damage of common pollution, with $\sigma > 0$; first introduced on p.13.

$\theta$ Baseline emission coefficient of industry-specific pollution, governing the scale of pollution generated per unit of output, with $\theta > 0$; first introduced on p.11.

# Appendix III. Assumptions

$Z_0$ Not too small, so as to avoid an initial phase in which working capital is zero.

$P_0$ Sufficiently small; for simplicity, normalized to zero.

$\theta \in (0,1)$ Higher-index industries are associated with lower emission intensity of industry-specific pollution.

$\varepsilon \in (0,1)$ Higher-index industries face greater difficulty in controlling industry-specific pollution.

$b > 1$ Households exhibit a preference for higher-index goods, net of industry-specific pollution.

$a > b + 1$ Ruling out the trivial case in which only the most capital-intensive good is produced, and ensuring that sector 0 is active when capital is sufficiently small.

$d = a$ Without loss of generality; otherwise, abatement effort can be rescaled proportionally.

$\delta_1 = \delta_2 = 1$ Without loss of generality, both depreciation rates are normalized to one.

$\eta > \rho$ Ensuring that capital accumulation is sufficient to sustain positive working capital.

$\eta < \rho/\alpha$ Preventing explosive growth.

$\gamma \geq \varepsilon$ Ensuring the feasibility of the Pigouvian tax.

$\gamma < 1$ Guaranteeing the strict monotonicity of the rental price of capital.

# Appendix IV. Canonical Models

All notation in this appendix is independent of that used in the main text.

## IV-1 AL model

Andreoni and Levinson (2001) propose a parsimonious static model for EKC.

Consider a representative agent with utility function $U = C - P$, where $C$ and $P$ are the consumption and pollution, respectively. One unit of consumption generates one unit of pollution. The agent can undertake abatement effort $E$

to reduce pollution by an amount $Q$. Hence, $P = C - Q$. Suppose $Q = C^\alpha E^\beta$, where $\alpha > 0, \beta > 0$ and $\alpha + \beta > 1$. Given income $Y$, the agent chooses $(C, E)$ to maximize his utility $U = C^\alpha E^\beta$ subject to $C + E = Y$, yielding the optimal allocation $C = \frac{\alpha}{\alpha+\beta}Y$, $E = \frac{\beta}{\alpha+\beta}Y$. It implies $P = aY - bY^{\alpha+\beta}$ for some positive constants $a, b$. Thus, the pollution-income relationship is inverted-U-shaped. This delivers the EKC.

## IV-2 Green Solow model

Brock and Taylor (2010) develop the Green Solow model, which extends the classical Solow framework by incorporating pollution and abatement activities.

Consider an one-sector economy in which the law of motion of capital $K$ is given by $\dot{K} = (1 - \theta)sY - \delta K$, where $\delta > 0$ is the constant depreciation rate, $s \in (0, 1)$ is the constant saving rate, $\theta \in (0, 1)$ denotes the constant share of total output $Y$ devoted to pollution abatement. Production function is $Y = K^\alpha(AL)^\beta$, where $L$ is the labor, $A$ is the labor-augmenting coefficient. Pollution emission is $E = \phi(\theta)BY$, where $B$ is the emission intensity, and $\phi(\cdot)$ is some function of abatement effort share. Suppose $\dot{L}/L = n \geq 0$, $\dot{A}/A = a \geq 0$, $\dot{B}/B = -b$, where $a, b, n$ are constants satisfying $b > a + n$. Let $k = K/(AL)$, $y = Y/L$, and assume $L(0) = A(0) = B(0) = 1$. Suppose further that $K(0) = K_0 > 0$ is sufficiently small.

Then, $\dot{k} = (1-\theta)sk^\alpha - (a+n+\delta)k$. Since $k(0)$ is sufficiently small, then, $k(t)$ is strictly increasing and converges to a steady state $k^* \in (0, \infty)$. Therefore, the time path of output per capita $y = Ak^\alpha$ is strictly increasing and tends to infinity. On the other hand, $\dot{E}/E = \alpha(1 - \theta)k^{-\beta} - [\alpha(a + n + \delta) + (b - a - n)]$, implying that the time path of $E$ is inverted-U-shaped. Thus, in the $(y, E)$ plane, $E$ is inverted-U-shaped with respect to $y$, yielding the EKC.

## IV-3 Stokey model

Stokey (1998) develops several models of growth and pollution, among which is the following one-sector growth model with accumulated pollution.

The social planner's problem is max $\int_0^\infty e^{-\rho t}(u(C) - X^\gamma)dt$, subject to $\dot{K} = Ae^{gt}K^\alpha z - \delta K - C$, $\dot{X} = Ae^{gt}K^\alpha z^\beta - \eta X$, $z \in [0, 1]$, and $K(0) = K_0 > 0$ and $X(0) = X_0 \geq 0$ are given. Here $X$ deotes pollution stock, $K$ capital stock, $z^{1/\alpha}$ the working capital share of the capital stock, $C$ consumption, and $u(C) = aC^{1-\sigma}$. Parameters $\rho > 0$, $\delta > 0$, $\eta \geq 0$, $a < 0$, $A > 0$, $g > 0$, $\alpha \in (0, 1)$, $\beta > 1$, $\gamma > 1$, $\sigma > 1$ are all constants. Using numerical simulations, she shows that the time path of $X$ is inverted-U-shaped, while output $x = Ae^{gt}K^\alpha z$ grows monotonically, yielding the EKC. She also discusses the use of Pigouvian taxation to implement the social optimum.

## IV-4 ES Model

Egli and Steger (2007) present a clever extension of the AL model to a dynamic setting. We further simplify it to the following version.

Consider a dynamic one-sector economy in which the unique final good is produced according to the linear technology $Y = rK$, where $Y$ denotes output, $K$ is the capital input, and $r > 0$ is a constant. A representative agent is endowed with an initial capital stock $K_0 > 0$ and has lifetime utility $\int_0^\infty e^{-\rho t} \ln U dt$, where $U = C - P$, $C$ and $P$ are consumption and pollution, respectively, and $\rho \in (0, r)$ is the subjective discount rate. One unit of consumption generates one unit of pollution. The agent can undertake abatement effort $E$ to reduce pollution by an amount $Q$, so that $P = C - Q$. Suppose that abatement is given by $Q = C^\alpha E^\beta$, where $\alpha > 0$, $\beta > 0$, and $\alpha + \beta > 1$, implying $U = C^\alpha E^\beta$.

The social planner chooses $(C, E)$ to maximize social welfare $\int_0^\infty e^{-\rho t} \ln U dt$, subject to $\dot{K} = rK - C - E$. It's easy to verify by Bellman equation that the unique optimal Markovian strategy is $C = \frac{\alpha\rho}{\alpha+\beta}K$, $E = \frac{\beta\rho}{\alpha+\beta}K$. This implies $P = aY - bY^{\alpha+\beta}$ for some constants $a > 0$ and $b > 0$, and $Y(t) = rK_0 e^{(r-\rho)t}$. Hence, income grows monotonically over time, and the pollution–income relationship is inverted-U-shaped, delivering an EKC in a dynamic setting.

## IV-5 JLW model

Ju, Lin, and Wang (2015) develop a model of industrial upgrading in which sectoral evolution is endogenously driven by changes in factor endowments.

Consider an economy consisting of three types of sectors. The first comprises a single industry producing capital goods; the second consists of a sequence of industries producing differentiated intermediate goods, indexed by $n = 0, 1, 2, \ldots$; and the third consists of a single industry producing a unique final good.

The production technologies of intermediate industries are given by $Y_0 = L_0$ and $Y_n = \min\left\{\frac{K_n}{a^n}, L_n\right\}$ for $n \in \mathbb{N}$, where $Y_n$ denotes output, $K_n$ and $L_n$ are capital and labor inputs, and $a > 1$ is a constant. Final output is produced according to $Y = Y_0 + \sum_{n=1}^{\infty} b^n Y_n$, where $b > 1$ is a constant.

Capital stock $Z$ evolves according to $\dot{Z} = \eta Z - K$, where $K$ is working capital used in production and constant $\eta > 0$ captures learning-by-doing effects. The representative agent is endowed with positive initial labor and capital and maximizes lifetime utility $\int_0^{\infty} e^{-\rho t} C^{\alpha} dt$, where $C$ is the final good consumption, $\rho > 0$ and $\alpha \in (0, 1)$.

Their main result can be summarized as follows. If $\rho < \eta < \rho/\alpha$ and $a > b + 1$, then, from the social planner's perspective, at any point in time at most two adjacent industries are active, while all others exit. Each industry exhibits a finite life cycle consisting of a growth phase and a decline phase, and industrial upgrading proceeds sequentially as capital accumulates. The authors further argue, invoking the Second Welfare Theorem, that the socially optimal allocation can be implemented by a competitive equilibrium.

## IV-6 MAG Model

Marsiglio, Ansuategi and Gallastegui (2016) propose a theoretical framework to study the EKC through the lens of structural change. Although the authors describe their framework as a two-sector growth model, the economy they analyze is, in essence, a one-sector growth model.

In their setup, there is a unique final good produced according to the production function $Y = a(uX)^{\alpha} K^{1-\alpha}$, where $Y$ denotes final output, $K$ is manufacturing capital, $X$ represents services (interpreted as a form of capital), and $u \in [0, 1]$ is the share of services allocated to final-good production. Capital and services accumulate according to $\dot{K} = a(uX)^{\alpha} K^{1-\alpha} - C$ and $\dot{X} = \theta(1-u)X + \phi X \dot{K}/K$, where $a > 0$, $\theta > 0$, $\phi > 0$, and $\alpha \in (0, 1)$ are constants. A representative household maximizes lifetime utility $\int_0^{\infty} e^{-\rho t} \big( bC^{\sigma} - \ln Z \big)\, dt$, subject to $\dot{K} = rK + p(uX) - C$, $\dot{X} = \theta(1-u)X + \phi X \dot{K}/K$, $u \in [0, 1]$, and pollution $Z = \eta K^{\varphi(1-\alpha)}$, where $b > 0$, $\sigma \in (0, 1)$, $\eta > 0$ and $\varphi > 0$ are constants, and $r$ and $p$ denote the rental rate of manufacturing capital and the price of services, respectively.

In this model, the parameter $\alpha$, measuring the share of services in the final-good production function, is interpreted as capturing the economic structure. Structural change is thus modeled as an exogenous increase in $\alpha$ over time. Under certain parametric conditions, the authors show that a monotonic increase in $\alpha$ along the development path may generate an inverted-U-shaped trajectory of pollution.

However, structural change in this framework is entirely exogenous. The model does not endogenously determine the allocation of resources across sectors, nor does it describe the joint evolution of sectoral composition and pollution dynamics. Despite being framed as a two-sector growth model, the economy effectively features a single final-good sector with two capital inputs. What the authors refer to as "structural change" is therefore not an endogenous reallocation of economic activity across sectors, but rather an exogenous change in factor shares within the production function. As a result, the model captures at most a compositional effect, rather than structural transformation in the strict sense.

## IV-7 AABH Model

Acemoglu, Aghion, Bursztyn, and Hemous (2012) introduce a two-sector model of directed technical change with environmental concerns. A simplified version is as follows.

Consider an infinite-horizon, discrete-time economy with a representative agent with preference $\sum_{t=0}^{\infty}(1+\rho)^{-t}u(C_t, S_t)$, where $\rho > 0$ is the constant discount rate, $C_t$ consumption of the unique final good at time $t$, $S_t$ environmental quality at time $t$, and $u$ an instantaneous utility function satisfying some regularity conditions. Final good is produced according to $Y_t = (Y_{ct}^{\sigma} + Y_{dt}^{\sigma})^{1/\sigma}$, where $Y_t$ is output, $Y_{ct}$ and $Y_{dt}$ are inputs of the clean and dirty goods, respectively, and $\sigma < 1$ is a constant. For each $j \in \{c, d\}$, sectoral output is given by $Y_{jt} = x_{jt}^{\alpha}(A_{jt}L_{jt})^{1-\alpha}$, where $\alpha \in (0, 1)$ is a constant, $L_{jt}$ is labor employed in sector $j$ at time $t$, and $A_{jt}$ and $x_{jt}$ are the quality and quantity of machines used in sector $j$, respectively. Each machine is produced using the final good according to $x_{jt} = aY_t$, where $a > 0$ is a constant.

At any time, there is a unit mass of scientists (entrepreneurs) who live for one period and must choose which sector to direct their research toward, but not both. A successful scientist invents an improved version of a machine that raises its quality by a factor $1+\gamma$ with constant $\gamma > 0$, and obtains a one-period patent. Each entrepreneur acts as a monopolist in machine production, using either the new technology upon successful innovation or the existing technology if innovation fails.

Let $s_{jt}$ denote the mass of researchers in sector $j$ at time $t$. The evolution of machine quality is given by $A_{jt} = (1 + \gamma\eta_j s_{jt})A_{j,t-1}$. It is assumed that the initial technology gap satisfies $A_{c0}/A_{d0}$ being sufficiently small.

Environmental quality evolves according to $S_{t+1} = \min\{\overline{S}, \max\{0, -\xi Y_{dt} + (1+\delta)S_t\}\}$, where $\overline{S} > 0$ is the maximum attainable environmental quality, and $\xi > 0$ and $\delta > 0$ are constants. An environmental disaster occurs if $S_t = 0$ for some finite $t$.

Their main results for the case in which the two inputs are substitutes can be summarized as follows. In the absence of government intervention, innovation occurs exclusively in the dirty sector, and an environmental disaster occurs with certainty. When the two inputs are sufficiently strong substitutes and $\overline{S}$ is sufficiently high, a temporary subsidy to clean research can prevent an environmental disaster. By contrast, when the two inputs are weak substitutes, a temporary subsidy to clean research cannot avert such a disaster. Finally, when the discount rate is sufficiently small, government intervention in the form of a Pigouvian tax on the dirty sector combined with a subsidy to the clean sector induces all innovation to switch to the clean sector in finite time. In this case, the optimal subsidy is temporary, and, moreover, if the two inputs are sufficiently strong substitutes, the optimal Pigouvian tax is also temporary.